Physical quantities

Physical quantities are divided into two groups

(a) Fundamental quantities

These are physical quantities which cannot be expressed in form of other quantities using any mathematical equations. They include

Quantity	S.I unit	Symbol of S.I unit
Mass	kilogram	kg
Time	second	s
Length	metres	m
Temperature	Kelvin	K
Current	Ampere	A

(b) Dimensional/derived quantities

These are physical quantities which can be expressed in terms of fundamental quantities. For example velocity, work, volume, density

Dimensions of physical quantities

This is a way in which derived quantities can be expressed in form of fundamental quantities. i.e.

Mass- M

Length- L

Time –T

The square bracket, [ ] is used to show dimensions

For example

(i) [Area] = length x length or length x width

$L \times L = L^2$

(ii) Volume = length x width x height

$= L \times L \times L = L^3$

(iii) Density = $\left[\frac{mass}{volume}\right] = \frac{M}{L^3}$ or ML^{-3}

(iv) Velocity = $\left[\frac{lenght}{time}\right] = \frac{L}{T}$ = LT^{-1}

(v) Acceleration = $\frac{velocity}{time}$ = $\frac{LT^{-1}}{T}$ = LT^{-2}

(vi) Force = mass x acceleration

= M x LT^{-2} = MLT^{-2}

Application of dimensions

(i) To check the validity of equation (equations that are not dimensionally consistent are obviously wrong expression and should be discorded.

(ii) To derive equations: for correct equation, the units of the left hand side must be similar to the units of the right hand side.

All right equations must be dimensionally consistent but not all dimensionally consistent equations are correct.

Examples 1

(a) The centripetal force F on a body of Mass M moving at constant speed V round a circular path of radius, r, is given by F = $\frac{MV^2}{r}$.
Show that the equation is dimensionally consistent

Solution
LHS = F = Ma = M x LT^{-2} = MLT^{-2}
RHS = $\frac{MV^2}{r}$. = $\frac{M\,x\,(LT^{-1})^2}{L}$ = MLT^{-2}
Since [LHS] =[RHS], the equation is dimensionally consistent.

(b) Show that the second equation of motion is dimensionally correct
s = ut + ½ at^2
[LHS] = [s] = L
[RHS]= [u][t] + ½ [a]$[T]^2$
=LT^{-1} x T + ½ LT^{-2} xT^2
= L + ½ L = $\frac{3}{2}L$
Since $\frac{3}{2}$ is a constant,
[LHS] =[RHS] showing that the equation is dimensionally consistent.

(c) The equation of a transverse wave of a rod of youngers modulus (E) and density, ρ, is given by v = $\sqrt{\frac{E}{\rho}}$. Show that it is dimensionally consistent.
Solution
[LHS] = [v] = LT^{-1}

[RHS]

E = ——

$E = \frac{stress}{strain} = \left[\frac{MLT^{-2}}{L^2}\right] = ML^{-1}T^{-2}$

$\sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}} = \sqrt{(L^2T^{-2})} = LT^{-1}$

[LHS] =[RHS] , the equation is dimensionally consistent

Derivation of equation

The method of dimensions can be used to derive equations.

Examples 2

The period (T) of a pendulum bob depends on the length of the pendulum (L), mass of the pendulum ball, M, and acceleration due to gravity, g. Determine an expression for the period of a simple pendulum T in terms of the quantities mentioned.

Solution

$T \propto MLg$

$T = kM^xL^yg^z$ where k is dimensionless constant

$[T] = [M]^x[L]^y[g]^z$

$[LHS] = [T] = T$

$[RHS] = M^xL^y[LT^{-2}]^z$

Equate powers of T, M,L

For T; -2z = 1

z = -½

For L; y + z =0

y – ½ = 0

y = ½

For M, x = 0

Therefore, $T = KL^{-\frac{1}{2}} \text{ x } g^{\frac{1}{2}} = K\sqrt{\frac{L}{g}}$

$T = K\sqrt{\frac{L}{g}}$

Exercise

1. The sphere of radius, α, moving through a liquid of density, ρ, and velocity, v, experiences a retarding force given by $F = k\alpha^x\rho^yv^z$, where K is a non dimensional constant. Use dimensions to find the values of x, y and z. [Ans, y =1, z =2 , y = 2]

2. Use dimensional analysis to show how the process of velocity transverse process vibration of a stretched string depends on its length, L, mass, m, and the tension F of the string $V = KL^xM^yF^z$, where k is a non-dimensional constant. Find the values of x, y, z. [Ans, x = ½ , y = - ½ , z = 1/2]
3. A cylindrical vessel of cross section area, A, contains air of volume, V, atmospheric

pressure, ρ, trapped by frictionless down and released.

If the piston oscillates with simple harmonic motion, show that the frequency is given by f = $\frac{A}{2\pi}\sqrt{\frac{\rho}{MV}}$ and show that the expression is correction.

4. The equation for volume, V, of a liquid flowing through a pipe in time, t, under steady flow id given by, $\frac{V}{t} = \frac{\pi r^4 \rho}{8\eta l}$
   r = radius of the pipe
   ρ = pressure difference between the two ends of the pipe
   l = length of the pipe
   η =coefficient of viscosity of the liquid
   If the dimensions of η are $ML^{-1}T^{-1}$ show that the above equation is dimensionally consistent.
5. For streamline flow of a non-viscous incompressible fluid , the pressure, ρ, at a point is related to height, h. and velocity, V, by the equation
   $(p-a)=pg(h-b) + \frac{1}{2} p(v^2 - d)$
   Where a, b, and d are constant and p is the density of fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent. Find the dimensions of a, b, and d.
   Solution
   Hint, we add or subtract quantities that have the same dimensions
   {ans. [a] has the same units as pressure = $ML^{-1}T^{-2}$, [b] has the same dimension as h = L, [d] has the same dimensions as $v^2 = L^2T^{-2}$}

Linear motion

Terms used

Displacement is the distance moved by a body in a specific direction

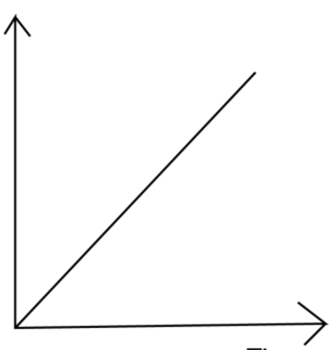
Velocity is the rate of change of displacement

Uniform velocity is the constant rate of change of displacement

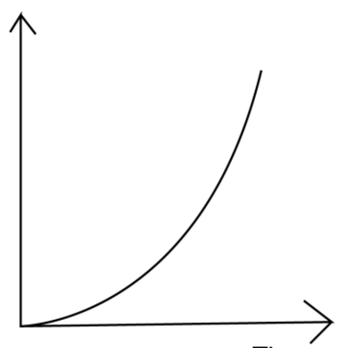
Acceleration is the rate of change of velocity.

Displacement time graphs

(i) Uniform velocity

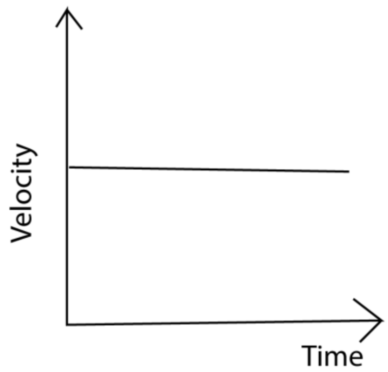


(ii) Uniform accelaration

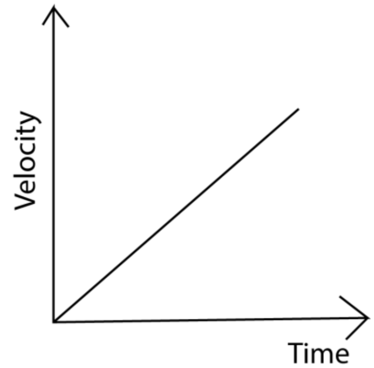


Velocity time graph

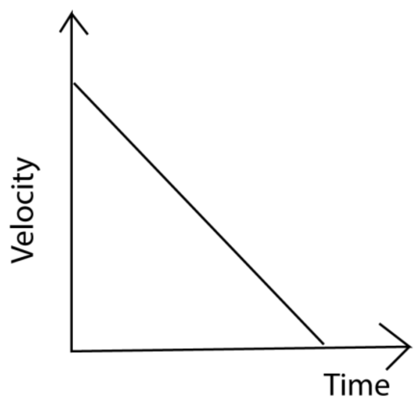
(i) Uniform velocity



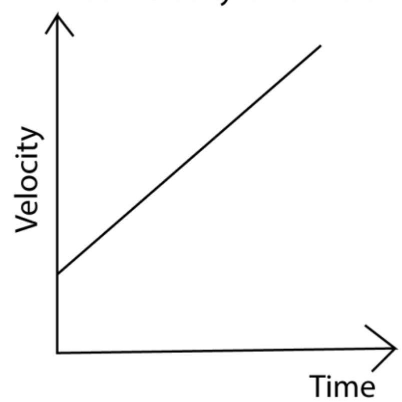
(ii) Uniform accelaration



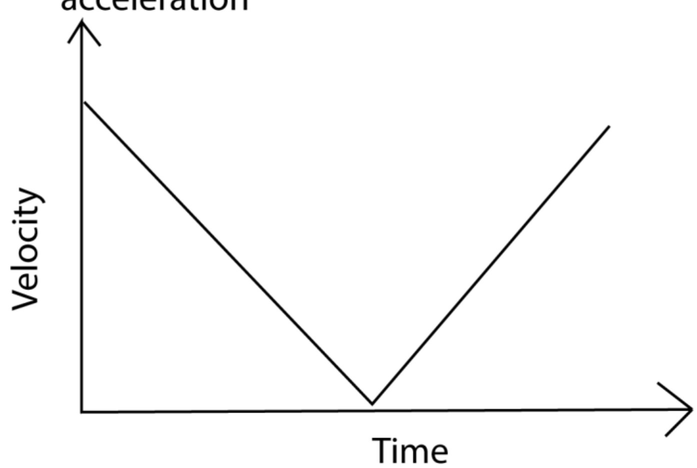
(iii) Uniform reterdation



(iv) Uniform accelaration where initial velocity is not zero



(v) For body thrown vertically upwards then falls afterwards with a uniform acceleration



Note: when velocity is uniform/constant or maximum, acceleration is zero

When the body starts from rest, initial velocity, u=0

When a body comes to the rest, the final velocity, v = 0

Equations of motion

Usual symbols

u – initial velocity

v- final velocity

s – displacement

a – acceleration

t – time

If a body's velocity changes from u to v in time t, then

(a) $\mathbf{a} = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t}$

or

v = u + at ……………………… (i)

As the body's velocity increases steadily

Average velocity $= \frac{v+u}{2}$

(b) Displacement, s = averages velocity x time

$= \frac{v+u}{2}t$

But v = u + at

$s = \frac{(u+at+u)t}{2}$

$s = \frac{(2u+at)t}{2}$

$s = ut + \frac{1}{2}at^2$ ………………… (ii)

From, $s = \frac{v+u}{2}t$

But v = u + at

$t = \frac{v-u}{a}$

$s = \frac{v+u}{2}\ x\ \frac{v-u}{a}$

$v^2 = u^2 + 2as$ ……………………(iii)

Example 1

A motorist travelling at constant speed of 50kmh^{-1} passes a motor cyclist starting of in the same direction. If the motorist maintains a constant acceleration of 2.8m/s^2,

(a) Calculate the time taken by the motorist to catch up with the motorist.

Motorist

u_1 =50km/hr $= \frac{50\ x\ 1000}{3600} = \frac{125}{9}ms^{-1}$

Distance moved by motorist, $s = \frac{125t}{9}\ m$

Where t = time taken by motorcyclist to catch up with the motorist

Distance s moved by motorcyclist = 0 + $\frac{1}{2}\ x\ 2.8t^2$

It implies that

. $\frac{125t}{9} = \frac{1}{2}\ x\ 2.8t^2$

t = 9.9s

(b) Speed of motorist as he overtakes the motorist
v = u + at = 0 + 9.9 x 2.8 = 27.8ms^{-1}

Examples 2

A motor car moving with uniform acceleration covers 5.5m in the 4th second and 9.5m in the 8th second in its motion. Find its acceleration and the initial velocity.

Solution

From s = ut + ½ at^2

Distance covered in 4th second = (distance covered in first 4seconds
– distance covered in first 3 second)

5.5 = (4u + ½ a x 4^2) – (3ut + ½ a x 4^2)

11 = 2u + 7a ……………………….. (a)

Distance covered in 8th second = (distance covered in first 4seconds
– distance covered in first 3 second)

9.5 = (8u + ½ a x 8^2) – (7ut + ½ a x 7^2)

19 = 2u + 15a ……………………….. (b)

Solving: a = 1ms^{-2}; u = 2ms^{-1}

Motion under gravity

In absence of any resistance, all bodies regardless of their mass fall with same acceleration near the earth's surface.

Acceleration due to gravity is the rate of change of velocity for freely falling body and it is symbolized as g. g = $9.81ms^2$ and it replaces "a" in equation of motion

For a body falling under gravity, g is positive and g is negative for the object moving upwards.

Example 1

A ball is thrown vertically upwards with initial speed $20ms^{-1}$. After reaching the maximum height and on the way down it strikes a bird 10m above the ground.

(a) Calculate the highest point reached

$u = 20ms^{-1}$, $g = -9.8ms^{-1}$, $v = 0$

from $v^2 = u^2 + 2as$

$0 = 20^2 + 2 \times -9.8 \times s$

The highest distance, s = 20.4m

(b) Calculate the speed at which it strike the bird

$u = 0$, $s = (20.4 - 10) = 10.4m$, $g = 9.8ms^{-2}$

from $v^2 = u^2 + 2as$

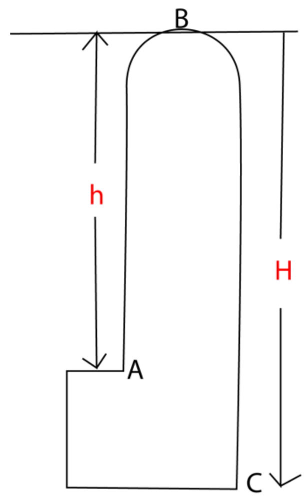
$v2 = 0^2 + 2 \times 9.8 \times 10.4$

$v = 14.3ms^{-1}$

Example 2

A stone is thrown vertically upwards with a speed of $10ms^{-1}$ from a building. If it takes 2.5 seconds to reach the ground, find the height of the building.

Solution



Time taken to move from A to B

$v = u - at$

$0 = 10 - 9.81t$; $t = 1.02s$

Height, h

$h = ut - \frac{1}{2} gt^2$

$= 10 \times 1.02 - \frac{1}{2} \times 9.81 \times (1.02)^2$

$= 5.1m$

Time taken from B to C = 2.5 – 1.02 =1.48s

Distance, H, u = 0, t = 1.48, g -9.81ms^{-2}

$H = 0 \times 1.48 + \frac{1}{2} \times 9.81 \times 1.48^2$

$= 10.7m$

Height of the building = H – h = 10.7-5.1 = 5.6m

Exercise

1 A ball is thrown straight upwards with a speed ams^{-1} from a point h m above the ground. Show that time taken to reach the ground is $t = \frac{u}{9}\left[1 + \left(1 + \frac{2gh}{u^2}\right)^{\frac{1}{2}}\right]$

2. A motorist travelling at a constant speed of 50 kmh^{1} passes a motorcyclist just starting off in the same direction. If the motorcyclists maintains a constant acceleration of 2.8ms^{-2} calculate; (i) Time taken by motorcyclist to catch up with the motorist. (9.9s)

(ii) The speed at which the motorcyclist overtakes the motorist.(27.72ms^{-1})

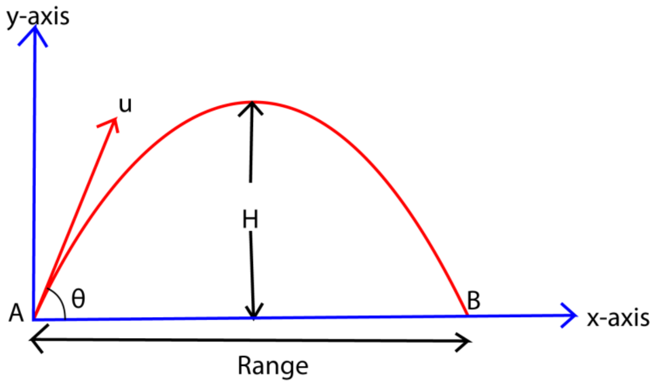
(iii) The distance travelled by the motorcyclists before overtaking.(137.2m)

3. A stone is thrown vertically upwards from a point at a height, h, above the ground level and initial velocity $20ms^{-1}$. If the stone, hits the ground, 5s later; find h [Answer 22.625m

Projectile

A projectile is anything which is given an initial velocity and left to move on its own in the presence of a constant force field, e.g. gravitation force field. In this case, air resistance is negligible.

Consider a body projected with a speed u at an angle θ to the horizontal



θ = angle of project

A –point of projection

H- maximum height of projection

AB –range

The projection has both vertical and horizontal component which are independent of each other. the acceleration due to gravity for the vertical component is g while that of the horizontal component is zero, that is, the horizontal velocity is constant.

Terminology

(a) **Angle of projection** is the angle between the direction of the projection and the horizontal.
(b) **Trajectory** is the path followed by a projectile
(c) **Maximum height, H**, is the distance between the highest point reached and the horizontal plane through the point of projection.
(d) **Time of flight (T)** is the time taken by the projectile or particle to move from its initial position to the final position along its path.
(e) **Horizontal range** is the distance from the initial to the final position of projection.

Horizontal motion

Horizontal component of velocity is got by

$v_x = u_x + a_x t$.

Where v_x, is the velocity of a body at any time t, while u_x and a_x are the initial component of velocity and horizontal acceleration respectively.

But $u_x = u\cos\theta$, since $a_x = 0$

Hence $v_x = u\cos\theta$ -------------(1)

From the above equation the horizontal velocity is constant throughout motion.

The horizontal distance , x, travelled after time t is given by

$x = u_x t + ½ at^2$

But $a_x = 0$

$\therefore x = u_x t \cos\theta$……………………(2)

Vertical motion

$v_y = u_y + a_y t$ where v_y, is the vertical velocity of a body at any time, t, while u_y and a_y are initial velocity component of velocity and vertical acceleration respectively.

$u_y = u\sin\theta, a_y = -g$

$v_y = v\sin\theta - gt$.......... …………………………………………………........(3)

The vertical displacement, y, is obtained below

$y = u_y t + ½ a_y t^2$

But $u_y = u\sin\theta$, $a_y = -g$

Hence

$y = (u\sin\theta)\, t - ½gt^2$……………………………………………...…………(4)

Speed, V, at any time t is given by

$v = \left[\sqrt{v_x^2 + v_y^2}\right]$……………………………………………………………(5)

The angle, α, the body makes with the horizontal after t is given by

$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$ ………………………………………………………… (6)

Maximum height, H

At maximum height, $v_y = 0$

$v_y^{2\,2} = u_y^2 + 2aH$

$0 = (u\sin\theta)^2 - 2ghH$

$H = \frac{u^2\sin^2\theta}{2g}$ ………………………………… … … … … … … … … … … … … … ….…….(7)

2 2

Time to reach the maximum heights

Using v = u + at

$0 = u_y + a_y t$

$0 = u\sin\theta - gt$

$t = \frac{u\sin\theta}{g}$ ……………………………………………………………………(8

Time of flight, T

The time taken by the projectile to move from the point of projection to a point on the plane through the point of projection where the projection lies i.e. time taken to move from A to B.

At B, y = 0

$y = ut\sin\theta - \frac{gt}{2}$

$0 = 2at\sin\theta - gt^2$

$0 = t(2u\sin\theta - gt)$

Either t =0 or $T = \frac{2u\sin\theta}{g}$ ………………………………………………….….(9)

Note: time of flight is twice the time taken to reach the maximum height

<u>Ranges , R:</u>

It is the distance between the point of projection and a point on the plane through the point of projection where the projectile lands i.e. horizontal distance AB.

$x = ut\cos\theta$

When x= R, $t = T = \frac{2u\sin\theta}{g}$

$\therefore R = u.\frac{2u\sin\theta}{g}\cos\theta = \frac{2u^2\sin\theta\cos\theta}{s}$ …………………………..….(10)

But $2\sin\theta\cos\theta = \sin 2\theta$

$R = \frac{u^2\sin 2\theta}{g}$

$For\ maximum\ range\ (R_{max})$

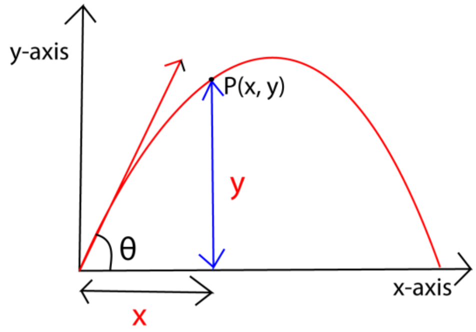
At R_{max}, $\theta = 45^0$

$\sin 2\theta = \sin 90$

$R_{max} = \frac{u^2}{g}$

Equation of trajectory

Consider a body project with a speed, u, from the ground and angle θ from horizontal.



Suppose the body passes through a point P(x, y) after time, t.

Consider vertical motion

x = (ucos θ)t

t = $\frac{x}{u\cos\theta}$

Consider vertical motion

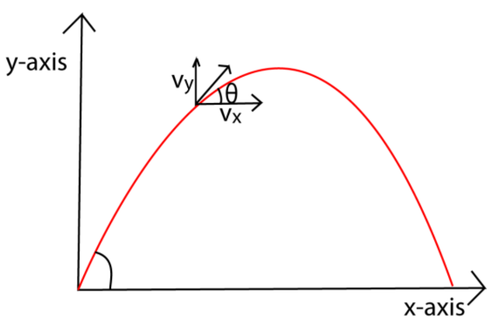
y = (usin θ)t – ½ gt^2

Substituting for t

y = (usin θ)$\frac{x}{u\cos\theta}$ – ½ g($\frac{x}{u\cos\theta}$)^2

= xtanθ - $\frac{gx^2}{2u^2}$(1 + tan^{2}θ)

Direction of motion



The direction of motion is determined by the direction of velocity of particles at any time, t. and its angle θ to which the velocity makes with the horizontal

$\tan\theta = \frac{v_y}{v_x}$

But v= u + at

v_y = usin θ -gt

$v_x = u\cos\theta$

$\theta = \tan^{-1}\left[\frac{u\sin\theta - gt}{u\cos\theta}\right]$

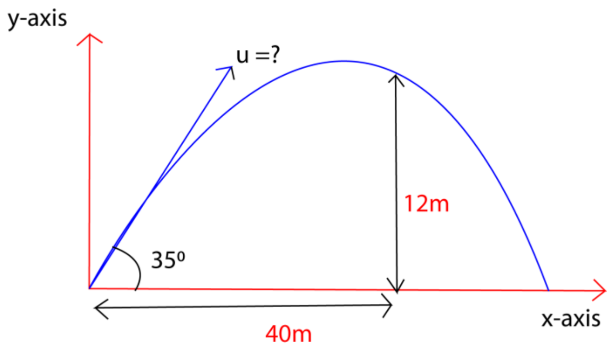
Magnitude of velocity, v= $\sqrt{[v_x^2 - v_y^2]}$

Example 3

A particle is projected at 35^0 to the horizontal and just clears a wall 12m high and 40m away from the point of projection. Find

(i) The speed of projection

(ii) Velocity of particle when it strikes the wall and time taken to reach the wall.



(i) From $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$

$12 = 40\tan 35 - \frac{9.81 \times 40 \times 40}{2u^2}(1 + \tan^2 35^0)$

$u = \sqrt{730.6085} = 27.03\text{ms}^{-1}$

(ii) From $t = \frac{x}{u\cos\theta} = \frac{40}{27.03\cos 35^0}$

t = 1.8s

Velocity, v

From v=$\sqrt{[v_x^2 - v_y^2]}$

$v_x = 27.03cos\ 35$ = 22.14ms^{-1}

v_y = 27.03sin 35^0 -9.81 x 1.8 = -2.658

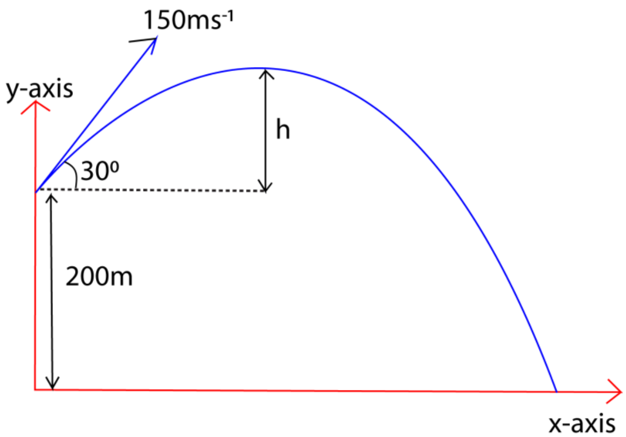
v = $\sqrt{[22.14^2 + (-2.658)^2]}$ = 22.3ms^{-1}

Example 4

A bullet is fired from a gun placed at a height of 200m with a velocity of 150ms^{-1} at an angle of 30^0. Find the

(i) Maximum height attained
(ii) Time taken for the bullet to hit the ground

Solution



(i) From h =$\frac{u^2 \sin^2\theta}{2g}$

h = $\frac{150^2 \sin^2 30}{2\ x\ 9.81}$ = 286.7m

H = 200 + h

= 200 + 286.7 = 486.7m

(ii) Let time taken be t

From y = (usin θ)t – ½ gt^2

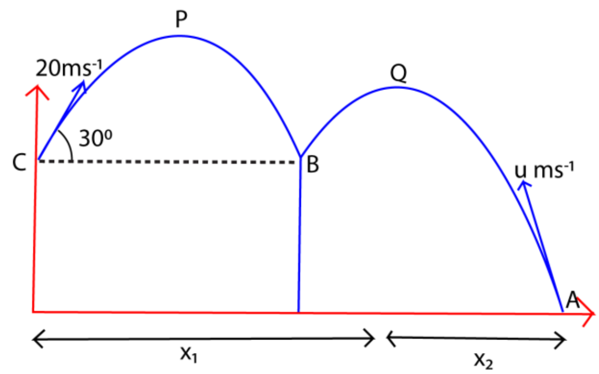
$-200 = (150\sin 30)t - \frac{1}{2} 9.81t^2$

$t = 17.6s$

Example 5

An object P is projected upwards from a height 60m above the ground from a height 60m above the ground with a velocity 20m/s at 30^0 to the horizontal, at the same time an object Q is projected from the ground upwards towards P at 30^0 to the horizontal. P and Q collided at a height of 60m above the ground. Find

(i) The speed of projection of the object Q.
(ii) The horizontal distance between the point of projection



(a) Speed of Q

Time of flight A and B = time of flight from C to B

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 30}{9.81} = 2.04s$$

Speed u of Q

(b) From $y = (u\sin\theta)t - \frac{1}{2} gt^2$

$60 = u\sin 30 \times 2.04 - \frac{1}{2} \times 9.81 \times (2.04)2$

$u = 78.84ms^{-1}$

(c) Range $x_1 = u\cos 30t$

$= 20 \times \cos 30 \times 2.04$

$= 35.33m$

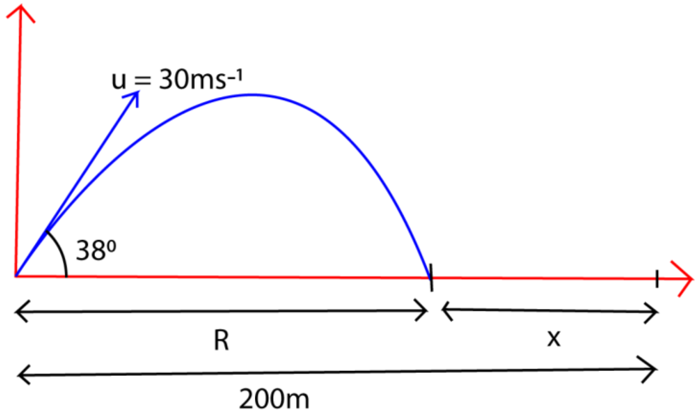
Distance $x_2 = 78.84 \times \cos 30 \times 2.04$

$= 139.29$

Distance between the point of projection = 139.29 + 35.33 = 174.62m

Example 6

Two foot ballers 120m apart standing facing each other, one kicks a ball from the ground such that the ball takes off at a velocity $30ms^{-1}$ at38^0 to the horizontal. Find the speed at which the second footballer must run towards the first footballer in order to trap the ball as it touches the ground if he starts running at the instant the ball is kicked.



Range, R $=\frac{u^2 sin2\theta}{g} = \frac{30^2\sin(2\ x\ 38)}{9.81}$ 89.02m

Time T taken to cover distance, R,

T $= \frac{2usin\theta}{g} = \frac{2usin\ \theta}{g} = \frac{2\ x\ 30x\sin 38}{9.81}$ =3.77s

Distance x to be covered by the second footballer = 120 – R = 120 – 89.02 = 30.98m

Speed $= \frac{Distance}{time} = \frac{30.98}{3.77} = 8.22ms^{-1}$

Exercise

1. A projectile is fired horizontally from the top of a cliff 250m high. The projectile landed 1.414 x 103m from the bottom of the cliff. Find the
   (i) Initial speed (Ans. $198.06ms^{-1}$)
   (ii) Velocity of the projectile just before it hits the ground. (Ans. $210.03ms^{-1}$)
2. (a) Define the term of flight and range as applied to the projectile motion.
   (b) A projectile is fired in air with a speed u ms-1 at an angle θ to the horizontal. Find the time of flight of the projectile.(T = $\frac{2usin\theta}{g}$)
3. (a) Define the term of flight and range as applied to the projectile motion.
   (b) A stone is projected at an angle 200 to the horizontal and just clears a wall which is 10m high and 30m from the point of projection. Find the
   (i) speed of projection [Ans. $73.75ms^{-1}$]
   (ii) Angle at which the stone makes with the horizontal at it clears the wall.[Ans. 16.84^0]
4. Prove that the time of flight T and the horizontal range, R, of a projectile are connected by equation, gT2 = 2Ttan α. Where α is the angle of projections.

5. A projectile is fired from ground level with a velocity of $500ms^{-1}$ at 30^0 to the horizontal. Find the horizontal range, the greatest height to which it rises and time taken to reach the greatest height. What is the least speed with which it could be projected in order to achieve the same horizontal range? [Ans. Range = 22069.96m, H = 3185.m, u_{min} =$465,3ms^{-1}$)
6. A body is thrown from the top of a tower 30.4m high with a velocity of $24ms^{-1}$ at an elevation of 300 above the horizontal. Find the horizontal distance from the roof of the tower of the point where it hits the ground. [Ans. 61.1m]
7. A body is projected at such an angle that the horizontal range is three times the greatest height. Given that the range is 400m, fins the necessary velocity of projection and angle of projection. [velocity = $64ms^{-1}$, angle = 53.13^0]
8. A projectile fire at an angle of 60^0 above the horizontal trikes a building 30 away at a point 5m above the point of projection. Find
   (i) The speed of projection. [time = 3.094s]
   (ii) Velocity of the projectile when it strikes the building.[u = $19.39ms^{-1}$]
9. An object P is projected upwards from a height of 60m above the ground with a velocity of $20ms^{-1}$ at 30^0 to horizontal. P and Q collide at a height 60m above the ground while they are both moving downward. Find
   (i) The speed of projection Q.[Ans. $78.84ms^{-1}$]
   (ii) The horizontal distance between the points of projection [174.62m]
   (iii) The kinetic energy of P before the collision with Q if the mass of P is 0.5kg [Answer200J]

Linear momentum

Linear momentum is a product of the body's mass and its velocity.

The S1 unit of momentum is $kgms^{-1}$

When a force F is applied to a body, it changes the body's velocity from u to v, the size of the force and the time for which it acts on a body.

From F = ma

$$a = \frac{v-u}{t}$$

$$F = \frac{m(v-u)}{t}$$

$$Ft = m(v-u)$$

Impulse of force is the product of force and the duration of its action or impulse is the change in momentum of the body which is acted on by the force

Example 1

A body of mass 3kg initially moving with a velocity of $5ms^{-1}$ is acted on by a horizontal force of 15N for 2s. Find the impulse and final speed.

Solution

Impulse = Ft

= 15 x 2

= 30N

Impulse = change in momentum

30 = m(v-u)

30 = 3(v-5)

$v = 15ms^{-1}$

Example 2

A tennis ball has a mass of 0.07kg. it approaches a racket with a speed of $5ms^{-1}$ and bounces off and returns to the way it come with a speed of $4ms^{-1}$. The ball is in contact with the racket for 0.2 seconds. Calculate

(i) The impulse given to the ball
(ii) The average force exerted on the ball by the racket.

Solution

(i) Impulse = Ft = m(v-u)
= 0.07(-4-5)
= -0.63Ns

(ii) $F = m\left[\frac{v-u}{t}\right] = \frac{0.63}{0.2} = 3.15N$

Collisions and principles of conservation of linear momentum

When two or more bodies collide, the total momentum of the system is conserved provided there is no external force on the sysytem.

Consider a body of mass m_1 moving with a velocity u_1 to the right. Suppose the body makes a head on collision with a nother body of mass m_2 moving with velocity u_2 in the same direction

Let v_1 and v_2 be the velocities of the 2 bodies respectively after collision

Let F1 be the force exerted on m_2 by m_1 and F_2 the force exerted on m_1 by m_2 using Newton's 2nd law.

$F_1 = m_1\left(\frac{v_1 - u_1}{t}\right)$, $F_2 = m_2\left(\frac{v_2 - u_2}{t}\right)$, where t is the time of collision

Using Newton's third law

$F_1 = -F_2$

$m_1\left(\frac{v_1 - u_1}{t}\right), = -m_2\left(\frac{v_2 - u_2}{t}\right),$

$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$

Hence, total momentum before collision = total momentum after collision, in other words momentum is conserved.

When two bodies collide, there is a short period of contact during which each exerts a force on each other at that instant, the force which each exert on each other is equal and opposite.

Types of collision

Collisions can be categorized as inelastic collision, perfectly inelastic, elastic or perfectly elastic collisions.

Elastic or perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
Kinetic energy is conserved	Kinetic energy is not conserved	Kinetic energy is not conserved
Linear momentum is conserved	Linear momentum is conserved	Linear momentum is conserved
Bodies separate after collision, e.g. collision of gas molecules	Bodies separate after collision e.g. a ball bouncing from a concrete floor	Bodies stick together and move with a common velocity. E.g. a trailer colliding with a saloon car.

Elastic collision

Momentum is conserved

$m_1v_1 + m_2v_2 = m_1u_1+m_2u_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ……………………………(i)

Kinetic energy is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1(u_1^2 - v_1^2) = m_2(u_2^2 - v_2^2)$ ……………………….. (ii)

Equation (i) ÷ (ii)

$$\frac{m_1(u_1-v_1)}{m_1(u_1^2-v_1^2)} = \frac{m_2(u_2-v_2)}{m_2(u_2^2-v_2^2)}$$

$$\frac{(u_1-v_1)}{(u_1-v_1)(u_1+v_1)} = \frac{(u_2-v_2)}{(u_2-v_2)(u_2+v_2)}$$

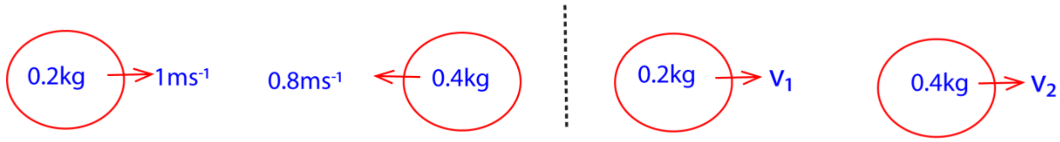
$$\frac{1}{(u_1+v_1)} = \frac{1}{(u_2+v_2)}$$

$(u_1 - u_2) = (v_2 - v_1)$

Example 3

A 200g block moves to the right at a speed of 100cms^{-1} and meets a 400g block moving to the left with a speed of 80cms^{-1}. Find the final velocity of each block if the collision is elastic.

Solution



$(v_2 - v_1) = -(-0.8 - 1)$

$(v_2 - v_1) = -1.8$ …………………… (i)

Using conservation of momentum

$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.2 \times 1) + (-0.4 \times 0.8) = 0.2v_1 + 0.4v_2$

$-0.12 = 0.2v_1 + 0.4v_2$

$-0.6 = v_1 + 2v_2$ ……………………..(ii)

Eqn (i) and Eqn (ii)

$v_2 = -1.8 + v_1$

$-0.6 = v_1 + 2(-1.8 + v_1)$

$V_1 = 1ms^{-1}$

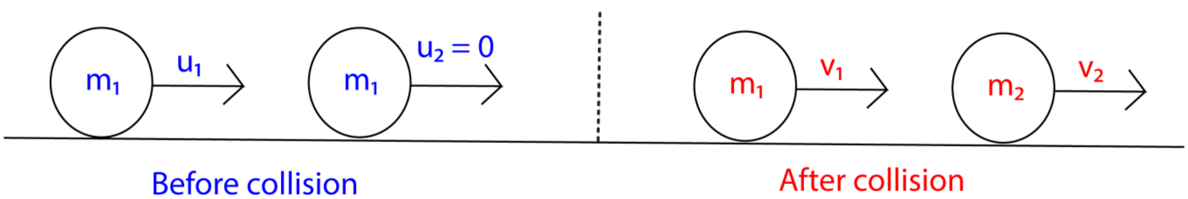
$V_2 = -0.8ms^{-1}$

Example 4

A particle of mass m_1, travelling with velocity u_1 makes a perfectly elastic collision with a stationary particle of mass m_2. After the collision, the first particle moves a velocity v_1 while the second particle moves in the same direction with velocity, v_2. Show that

$v_2 = \frac{2m_1u_1}{m_1 + m_2}$ and $v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)}$

Solution



From the principle of conservation of momentum

Momentum before collision = momentum after collision

$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$

$v_1 = \frac{m_1u_1 - m_2v_2}{m_1}$ ……………………………(i)

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2 x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$ ………………………………… (ii)

Substituting equation (i) into equation (ii)

$$m_1u_1^2 = m_1\left[\frac{m_1u_1 - m_2v_2}{m_1}\right]^2 + m_2v_2^2$$

$$m_1^2u_1^2 = m_1^2u_1^2 - 2m_1u_1m_2v_2 + m_2^2v_2^2 + m_1m_2v_2^2$$

$$2m_1u_1m_2v_2 = m_2^2v_2^2 + m_1m_2v_2^2$$

Dividing through by m_2v_2

$$2m_1u_1 = v_2(m_1 + m_2)$$

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)}$$

From the principle of conservation of momentum

Momentum before collision = momentum after collision

$$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$$

$$m_2v_2 = m_1(u_1 - v_1)$$

$$v_2 = \frac{m_1((u_1 - v_1)}{m_2}$$

$$v_2^2 = \frac{m_1^2}{m_2^2}(u_1 - v_1)^2 \text{ ....(i)}$$

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$v_2^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2) \text{ .... (ii)}$$

Equation (i) and (ii)

$$\frac{m_1^2}{m_2^2}(u_1 - v_1)^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2)$$

$$\frac{m_1}{m_2}(u_1 - v_1) = (u_1 + v_1)$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1$$

$$u_1(m_1 - m_2) = v_1(m_2 + m_1)$$

$$v_1 = \frac{u_1(m_1 - m_2)}{(m_2 + m_1)}$$

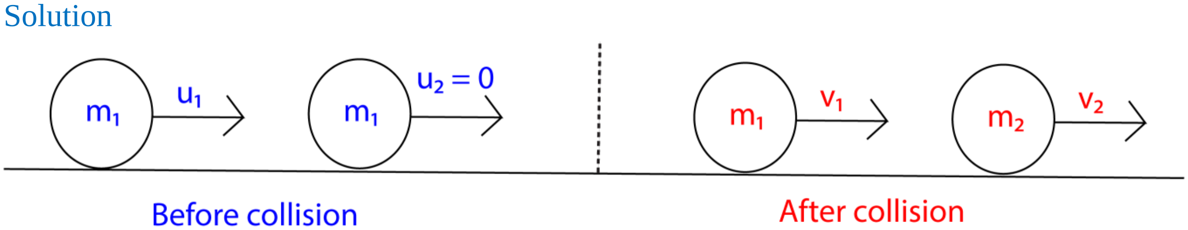
Example 5

A particle P of mass m_1, travelling with a speed u_1 makes a head on collision with a stationary particle Q of mass m_2. If the collision is elastic and speed of P and Q after impact are v_1 and v_2 respectively show that if b = $\frac{m_1}{m_2}$

(i) $\frac{u_1}{v_1} = \frac{b+1}{b-1}$

(ii) $\frac{v_2}{v_1} = \frac{2b}{b-1}$

Solution



From the principle of conservation of momentum

Momentum before collision = momentum after collision

$$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$$

$$v_2 = \frac{m_1(u_1 - v_2)}{m_2}$$

$$v_2 = b(u_1 - v_1) \text{ ....(i)}$$

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$v_2^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2)$$

$$v_2^2 = b(u_1^2 - v_1^2) \text{ .... (ii)}$$

Squaring equation (i)

$$v_2^2 = b^2(u_1 - v_1)^2 \text{ ....(iii)}$$

Equation (ii) and (iii)

$$b(u_1^2 - v_1^2) = b^2(u_1 - v_1)^2$$

$$(u_1 + v_1) = b((u_1 - v_1)$$

$$u_1(b-1) = v_1(b+1)$$

$$\frac{u_1}{v_1} = \frac{(b+1)}{(b-1)} \text{ .... (iv)}$$

(ii) Consider $\frac{u_1}{v_1} = \frac{(b+1)}{(b-1)}$

$$u_1 = v_1\frac{(b+1)}{(b-1)} \text{ ....(v)}$$

Substitution Eqn (v) into (i)

$$v_2 = b\ (v_1\frac{(b+1)}{(b-1)} - v_1)$$

Dividing through by v_1

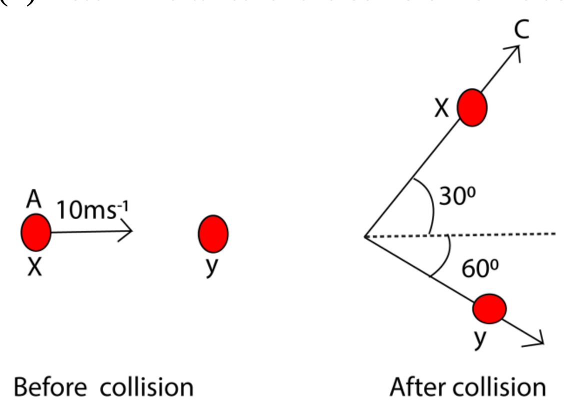
$$\frac{v_2}{v_1} = \frac{2b}{b-1}$$

Example 6

An object X of mass m moving with velocity $10ms^{-1}$ collides with a stationary object Y of equal mass. After collision, x, moves with speed u at an angle 30^0 to its initial direction, while Y moves with a speed of V at an angle 90^0 to the new direction of x.

(i) Calculate the speed u and v.

(ii) Determine whether the collision is inelastic or not.



Consider horizontal momentum

From the principle of conservation of conservation of momentum

m x 10 + m x 0 = m x ucos 30^0 + mvcos 60^0

$$10 = u\frac{\sqrt{3}}{2} + \frac{v}{2} \text{ ....(i)}$$

Consider vertical momentum

m x 0 + m x 0 = $\frac{u}{2} - v\frac{\sqrt{3}}{2}$

$$u = v\sqrt{3} \text{ ....(ii)}$$

Putting Eqn (ii) into Eqn (i)

$20 = (v\sqrt{3}\ x\ \sqrt{3} + v$

20 = 4v

v = $5ms^{-1}$

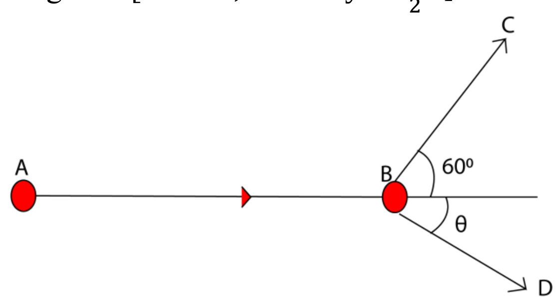
u = $5\sqrt{3}$ ms^{-1}

(iii) Kinetic energy before = $\frac{1}{2}\ x\ m\ x\ (10)^2$ = 50M J

Kinetic energy after collision =

Exercise

1. A bullet of mass 300g travelling at a speed of $8ms^{-1}$hits a body of mass 450g moving in the same direction as the bullet at 1.5ms-1. The bullet and the body move together after collision. Find the loss in kinetic energy. [Ans. 3.8025J]
2. A particle A of mass 4kg is incident with velocity V on a stationary helium nucleus B of mass 4kg. After collision, A moves in direction BC with velocity v/2 where BC makes an angle of 600 with the initial direction of AB and the helium nucleus moves along BD. Calculate the angle made in direction AB and the velocity of the helium along BD. [θ = 30^0, velocity = $\frac{v\sqrt{3}}{2}$]



3. (a) State Newton's laws of motion
   (b) Use Newton's laws of motion to show that when two bodies collide, their momentum is conserved.
   (c) Two balls P and Q travelling in the same direction line in opposite direction with speeds $6ms^{-1}$ and $15ms^{-1}$ inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the
   (i) final velocity [Ans. $2.08ms^{-1}$]
   (ii) change in kinetic energy [Ans. 28.03J]
   (d) (i) What is an impulse of force?
   (ii) Explain why a long jumper should normally land on sand.
   The force exerted on a long jumper on coming to rest is given by F = change in momentum over time taken. Since the change in momentum is constant, it Implies that if the time taken to when coming to rest is increased, then the force exerted on

the knees of the jumper reduces. Sand increases the time taken for the jumper to stop reducing the damaging force to the knee.

4. (a)(i) State the law of conservation of linear momentum
   (ii) Use Newton's laws to derive the above.
   (b) Distinguish between elastic and inelastic collisions
   (c) An object X of mass m moving with velocity 10ms-1 collides with a stationary object Y of equal mass. After collision, x, moves with speed u at an angle 30^0 to its initial direction, while Y moves with a speed of V at an angle 90^0 to the new direction.
   (i) Calculate the speed u and v. [Ans. u = $5\sqrt{3}$ ms^{-1}, v =$5ms^{-1}$]
   (ii) Determine whether the collision is inelastic or not. [Ans. kinetic energy on both sides = (50M)J, since kinetic energy is conserved, the collision is elastic]
5. (a) (i) Define linear momentum
   (ii) State the law of conservation of linear momentum.
   (iii) Show that in (ii) above follows Newton's laws of motion.
   (iv) Explain why, when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest.
   (b) A car of mass 100kg travelling at uniform velocity of 20ms-1 collides perfectly in elastically with a stationary car of mass 1500kg. Calculate the loss in kinetic energy of the car as a result of collision. [Ans. 168000J]

Moments

This is a product of the **force** multiplied by the perpendicular distance between its line of action and the axis of rotation.

The moment is also known at turning force or torque

Moment (Nm) = Force (N) x perpendicular distance (m)

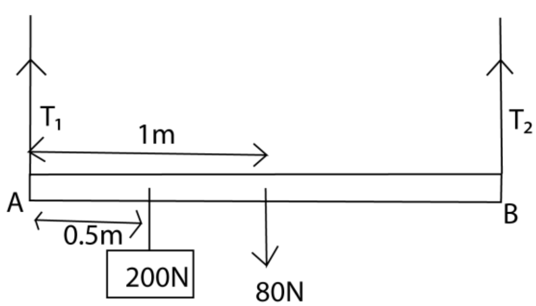
A couple
Two forces that are equal in magnitude but opposite in direction (and acting along parallel lines), thus creating the turning effect of a torque or moment.

Principle of moments

For an object to be in equilibrium (with no movements about a turning point), the sum of anticlockwise moments is equal to the sum of clockwise moments.

Examples 1
A rod mass 8kg and 2m long is balance horizontally by two inextensible string tied to the end A and B of the rod when a mass of 20kg hangs 0.5m from A. Find the tensions in the strings at A and B. [take g = $10ms^{-2}$]

Solution

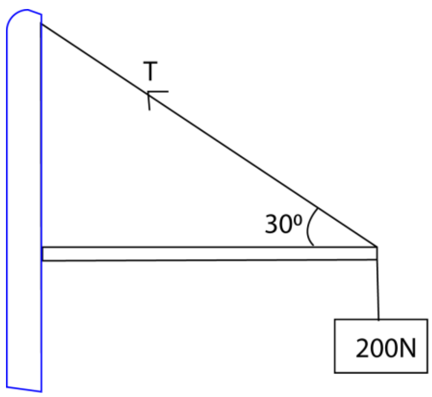


The sum of moments of initial at A = 0
$0 = 0.5 \times 200 + 1 \times 80 - T_2 \times 2$
$T_2 = 90$

But $T_1 + T_2 = 200 + 80$
$T_1 + 90 = 280$
$T_1 = 190N$

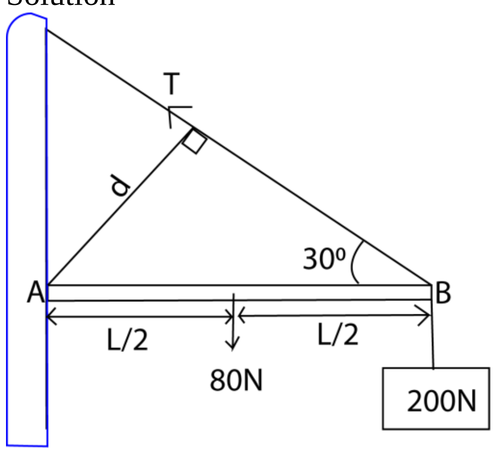
Example 2

A force of 200N hangs on a uniform rod of weight 80N and held at equilibrium by a string as shown in the figure below.



Calculate the tension in the string

Solution



Taking moments at A

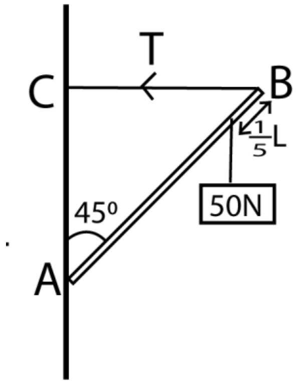
$T \times d = 80 \times \frac{L}{2} + 200L$

But $d = L \sin 30$

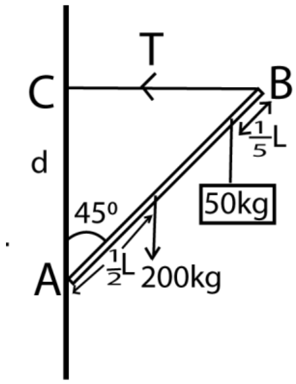
$TL\sin 30 = 240L$
$T = 480N$

Example 3

A rod of length L weighing 200kg is held by a horizontal string BC in equilibrium with a mass of 50kg as shown in the diagram below. Find the tension T.



Solution



Moments about A

T x d = (200 x 9.8) x $\frac{1}{2}Lcos\ 45$ + (50 x 9.8) x $\frac{4}{5}Lcos\ 45$

But d = Lcos 45

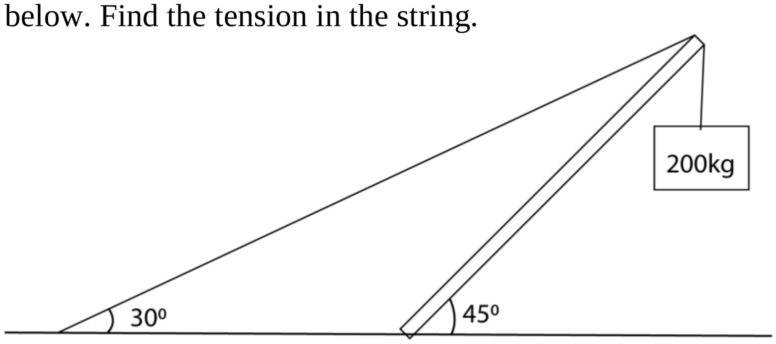
Therefore,

T x Lcos 45 = (200 x 9.8) x $\frac{1}{2}Lcos\ 45$ + (50 x 9.8) x $\frac{4}{5}Lcos\ 45$

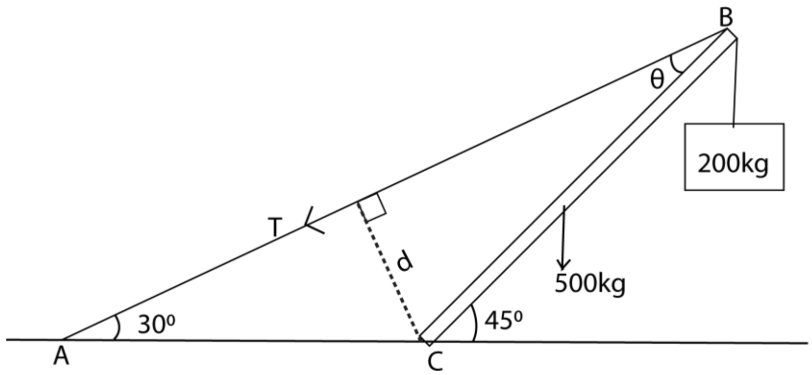
T = 1372N

Example 4

A rod of 500 N is balanced by a string AB and a mass of 200kg as shown in the diagram below. Find the tension in the string.



Solution



Taking moments at C
Let the length of the rod b L

T x d = (500 x 9.8) x $\frac{L}{2}$cos 45 + (200 x 9.8) x Lcos 45

But d = Lsinθ
$45^0 = 30^0 + \theta$
$\theta = 15^0$

It implies that
T x Lsin 15 = (500 x 9.8) x $\frac{L}{2}$cos 45 + (200 x 9.8) x Lcos 45
T =12047N

Exercise

1. A mass of 5kg is suspended from end A of a uniform bean of mass 1kg and length 1m. the end B of the beam is hinged to a wall. The beam is kept horizontal by a wire attached to point A and C on the wall at a height 0.75m above B.
   (i) Draw a sketch diagram to show the forces acting on the beam
   (ii) Calculate the tension in the wire. [T = 90N]
   (iii) What is the force exerted by hinge on the beam [F = 72.1N]
2. A uniform ladder AB of length 12m is placed at an angle of 60^0 to the horizontal with one end B leaning against the wall and the other end A on the ground. Calculate the reaction force R of the wall at B and force F of the ground at A if the weight of the ladder AB is 200N. [R = 200N, F = 57.7N]

Center of gravity

This is a point where the resultant force of attraction of a body acts

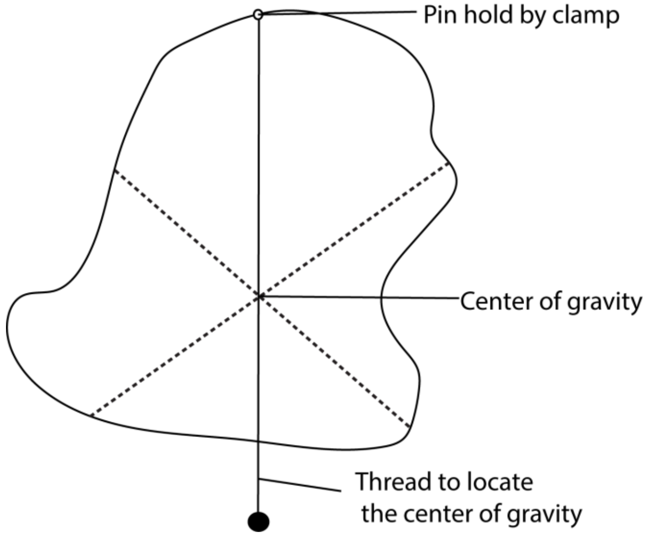
Center of gravity of regular object

For regular shape bodies, the center of gravity is at the geometric center of the body e.g. the center of gravity of a Uniform meter ruler is at 50cm mark, for circle, it is at the center. For a rectangular and square body it is at the point of intersection of the diagonals.

Irregular body

The best way of finding the center of gravity of an irregular object is by use a plumb line. A plumb line is made from a thread of cotton with a loop at one end and a weight tied at other end.

For a irregular card board for instance, three small holes are made at well-shaped intervals around the edge of the card



A pin is then put through one of the holes and firmly by a clamp and stand so that the card board swings on it.

The card board will come to rest with its center of gravity below the point of support along the vertical line of plumb line.

The cardboard is hung through another hole, the point of interception of the two vertical lines is the center of gravity.

Factors that affect stability

1. The position of center of gravity, should be low.
2. Width of the base: the wider the width of the base, the more stable the body is.

Way of increasing stability

1. Increasing the base area
2. Lowering the center of gravity

Application of center of gravity

1. cars have very heavy framed to lower center of gravity
2. Racing cars have wide wheel base to lower center of gravity.

Friction
Solid friction

Friction is the forces which oppose the relative motion of two surfaces in contact.

The direction of the friction force is opposite to the direction of motion of the body.

Types of friction

There are 2 types of friction i.e.

(i) Static friction

(ii) Kinetic friction / sliding friction

Static friction opposes the tendency of one body sliding over the other.

Kinetic/sliding/dynamic friction opposes the sliding of one body over the other.

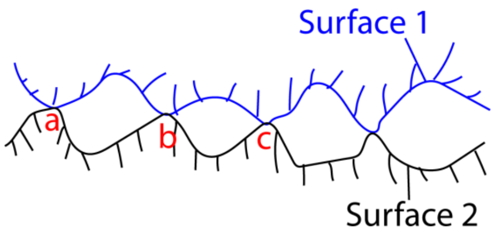
Limiting friction is the maximum friction between on two surfaces.

Laws of solid Friction

1. The frictional force between two surfaces opposes their relative motion.
2. The frictional force is independent of the area of contact of the given surface when the normal reaction is constant.
3. The limiting frictional force is proportional to the normal reaction for case of static friction. The frictional force is proportional to the normal reaction for the case of kinetic (dynamic) friction and is independent of the relative velocity of the surfaces

Molecular Theory and the laws of solid friction

On a microscopic level, even a highly polished surface has bumps and hollow. It follows that when 2 surfaces are put together, the actual area of contact is less than the apparent area of contact.



At points of contact like a, b, c, small cold-welded joints are formed by the strong adhesive forces between the molecules in the two surfaces.

These joints have to be broken before one surface can move over the other.

This accounts for law 1.

The actual area of contact is proportional with the normal force (reaction). The frictional force which is determined by the actual area of contact at the joints is expected to be proportional to the normal force.

This accounts for law 1 and 3

If the apparent area of contact of the body is decreased by turning the body so that it rests on one of the smaller side, the number of contact points is reduced. Since the weight of the body has not altered, there is increased pressure at the contact points and this flattens the bumps so that total contact area and the pressure return to their original values.

Therefore, although the apparent area of contact has been changed, the actual area of contact has not.

This accounts for law 3

Coefficient of static friction

Coefficient of limiting friction is proportional to the normal reaction or it weight.

i.e. $\frac{limiting\ fricitional\ force\ (F)}{normal\ reaction\ (R)} = \mu, a\ constnat$

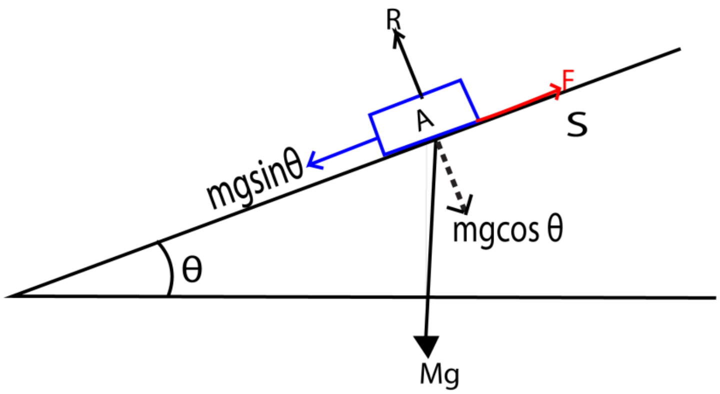
μ is known as the coefficient of friction between the two surfaces. The magnitude of μ depends on the nature of the two surfaces; for example it is about 0.2 to 0.5 for wood on wood, and about 0.2 to 0.6 for wood on metals.

Measurement of coefficient of static friction, μ_s

Method 1: Using a tilting plane.

A block A is placed on a plane and the plane is tilted until when the block begins to slide. The angle of θ of inclination of the plane surface to the horizontal is measured.

The co-efficient of friction is given by $\mu_s = \tan\theta$



When the block is at the point of sliding

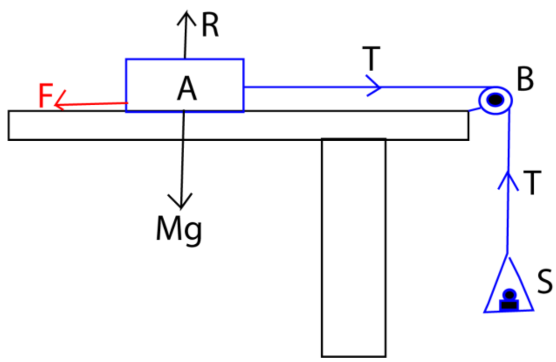
Fs = Wsinθ ………….(i)

R = Wcosθ…………..(ii)

(i) ÷ (ii)

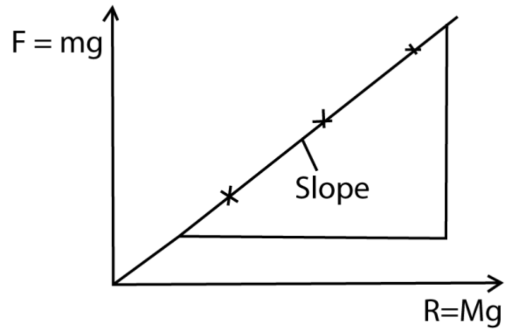
$\frac{Fr}{R} = \frac{Wsin\,\theta}{Wcos\,\theta} = \mu_s = \tan\theta$

Method 2: To determine the co-efficient of static friction.



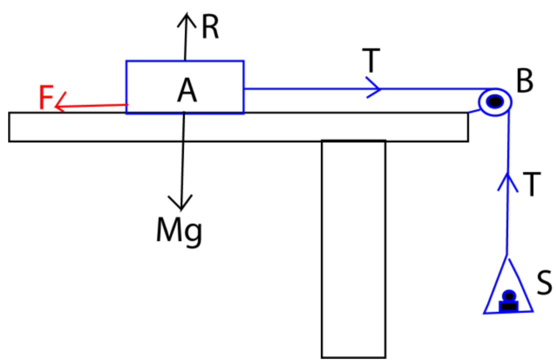
Masses are added to the scale pan until the block just slides. The total mass m of the scale pan and masses added is noted. The procedures are repeated for different values of R obtained by adding known weights to the block.

A graph of mg against R(Mg) is plotted.



The slope of the graph is μ_s

Co-efficient of kinetic (dynamic friction



A wooded block of known weight is connected to a scale pan by a string passing over a smooth pulley as shown in the diagram above.

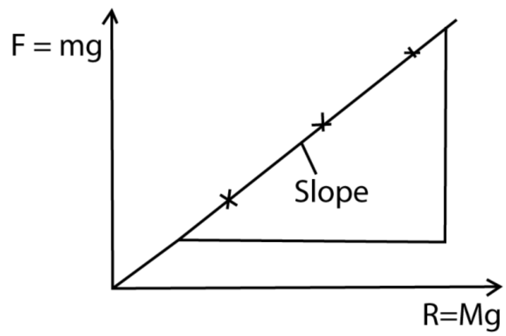
Small masses are added to the scale pan one at a time and each time the block is given a slight push. This is repeated until the block moves with a constant speed after a light push.

At this instant, the weight of the scale pan and all masses added to it is equal to kinetic frictional force (f).

The weight of the block is equal to the normal reaction. the normal reaction is varied by adding known weights on the block at each time finding the corresponding frictional force.

The values of the normal reaction R and corresponding kinetic force (f) is recorded in the table.

A graph of F against R is plotted and its slope gives the coefficient of kinetic friction

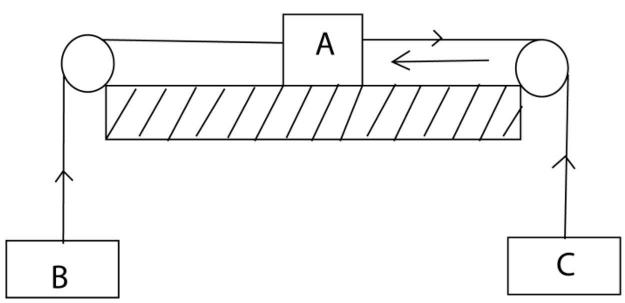


The slope of the graph is μs, is coefficient of kinetic friction

Advantage of friction
- Used in writing
- Used in movement
- Used in walking

Disadvantage of friction
- Wears machines
- Wears shoes
- Causes unnecessary noise in moving parts of a machines.

Examples 1

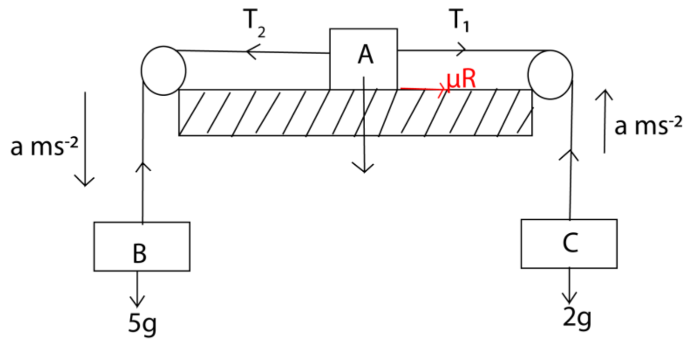


A, B, C are particles with masses 7, 5, 2kg respectively. when the system is released from rest with coefficient of friction 1/5.

Find

(a) Acceleration of the system
(b) Tension of the strings
(c) Distance moved by B after 4s

Solution



(a) Consider B

$5a = 5g – T_2$ ……………………………..(i)

Consider C

$2a = T_1 -2g$…………………………… (ii)

Consider A

$7a = T_2 – (T_1 + \mu R)$

But R = 7g

$7a = T2 – T1 - \frac{1}{5}\ x\ 7g$ ………………. (iii)

Eqn (i) + Eqn (ii) + Eqn (iii)

$5a + 2a + 7a = 5g – T_1 + T_2 – 2g + T_1 - T_2 - \frac{1}{5}\ x\ 7g$

$14a = 1.6g$

$a = \frac{1.6\ x\ 9.81}{14} = 1.12ms^{-2}$

(b) From (i)

$5a = 5g – T_2$

$5 \text{ x } 1.12 = 5 \text{ x } 9.81 – T_2$

$T_2 = 43.45N$

From (ii)

$2a = T_1 -2g$

$2 \text{ x } 1.12 = T1 – 2g$

$T1 = 21.86N$

(c) $s = ut + \frac{1}{2}at^2$

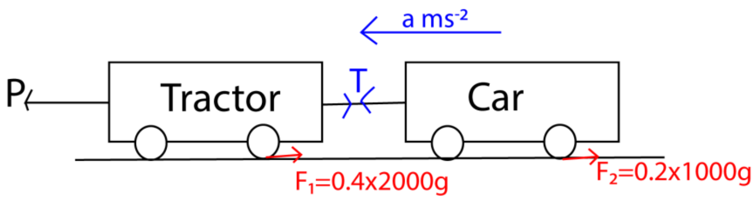
$s = 0 \text{ x } 4 + 1.12 \text{ x } 4 \text{ x } 4 = 17.92m$

Example2

A tractor of mass 2000kg is used to pull a car of mass 1000kg to which it is connected by a chain whose mass can be neglected. The tractor pulling steadily moves the car from rest along a horizontal road through a distance of 12,5m in 5s. The coefficient of kinetic friction between the tyres of the tractor and the road is 0.4 and that between the tyres of the car and the road is 0.2.

Find the pull exerted by the tractor’s engine.

Solution



Consider motion of the tractor

$2000a = P- (T + F_1)$

$= P – T- 0.4x\ 2000 \times 9.81$

$= P – T – 7848$ ……………………………………(i)

Consider motion of the tractor

$1000a = T – F_2$

$= T – 0.2 \times 1000 \times 9.81$

$= T – 1962$ …………………………………….. (ii)

Eqn (i) + Eqn (ii)

$3000a = P – 9810$

From $s = ut + \frac{1}{2}t^2$

$12.5 = 0 \times 5 + \frac{1}{2} \times 5 \times 5$

a = 1ms-2

thus, P = 3000 x1 + 9810 = 12810N

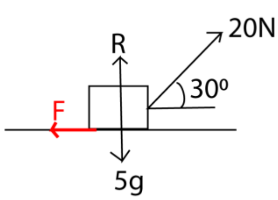
Example 3

A body of mass 5kg is at rest on a rough horizontal plane of coefficient of friction of 0.6. a force of 20 N at 300 above the horizontal is applied on the body. Find

(i) normal reaction

(ii) frictional force exerted by the floor on the body.

Solution



(i) Resolving vertically

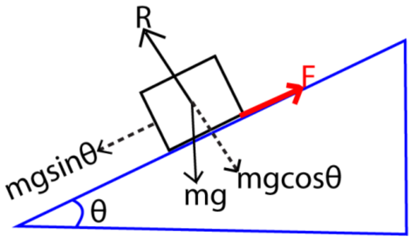
$R + 20\sin 30^0 = 5g$

$R = 5 \times 9.81 – 10 = 39.05N$

(ii) $F = \mu R = 0.6\ 39.05 = 23.43N$

Motion on an inclined plane

(i) When a body is moving down the slope



Resulting force = mgsinθ –F

ma = mgsinθ –F

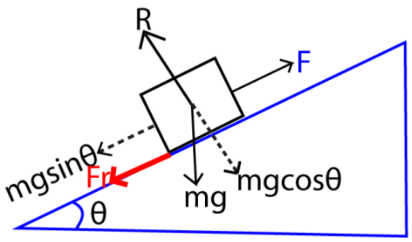
but F = μR

and R = mgcosθ

⇨ ma = mgsinθ –μcosθ

a = g(sinθ –μcosθ)

(ii) when the body is moving up the slope



Resultant force = F + mgsinθ

ma = mgsinθ + F

but F = μR

and R = mgcosθ

⇨ ma = mgsinθ +μcosθ

a = g(sinθ + μcosθ)

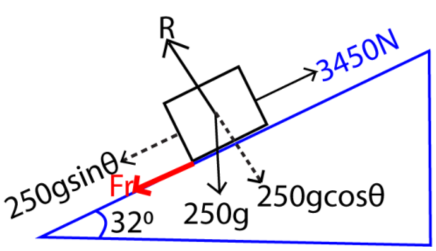
Example 4

A car of mass 0.25 x 10^3kg ant a tractive pull of 3450N climbs a truck which is inclined at 32^0 to the horizontal. The velocity of the car at the bottom of the inclined plane is 27ms^{-1} and the coefficient of friction between the plane and the car tyres is 0.25. Calculate

(a) distance travelled along the inclined before the car comes to rest.

(b) Time taken before the car comes to rest

Solution



(a) From $ma = F - (f + mg\sin\theta)$, $f = \mu R = \mu mg\cos 32^0$

$250a = 3450 - (0.25 \times 250 \times 9.81\cos 32^0 + 250 \times 9.81\sin 32^0)$

$a = 6.52ms^{-2}$

From $v^2 = u^2 - 2as$

$0 = 27^2 - 2 \times 6.52s$

$s = 55.9m$

from $v = u - at$

$0 = 27 - 6.52t$

$t = 4.15s$

Exercise

1. A car of mass 200kg moving along a straight road at a speed of $96kmh^{-1}$ is brought to rest by steady application of the brakes in a distance of 80m. Find the co-efficient of kinetic friction between the tires and the road. [hint $ma = \mu mg$; μ=0.45]

2. A car of mass 1.5×10^3kg and tractive pull 3.5×10^3N climbs a truck which is inclined at an angle of 30^0 to the horizontal. The speed of the car at the bottom of the incline is $20ms^{-1}$ and the coefficient of sliding friction is 0.25, calculate
   (i) The distance travelled along the incline before the car comes to a halt.
   (ii) The time taken travelling along the incline before the car comes to a halt.
   [Ans. s = 42.6m, t = 4.26s]

3. An old car of mass 1500kg and tractive pull 4000N climbs a tract which is inclined at an angle of 30^0 to the horizontal. The velocity of the car at the bottom of the incline is $108kmh^{-1}$ and the coefficient of sliding friction is 0.35.
   (i) Calculate the distance travelled along the incline before the car comes to a halt.(86.53m)
   (ii) The time taken to travel along the incline before the car comes to a halt.(5.77s)

4. In an experiment to determine the coefficient of static friction between a block and a plane, a student placed the block on a wooded surface and tilted the surface until the block just began to move. He observed that this happened at an angle of inclination of the plane with the horizontal of 20^0 and the block slid 100cm down the plane in 2s. Calculate the coefficient of static friction. [μ = 0.31]

5. Two masses m_1 and m_2 rests on a rough faces of a double inclined plane and connected by a light inextensible string passing over a pulley at the top of the plane. If m1>m2, show that acceleration of the system $a = \frac{g[m_1(\sin\alpha - \mu\cos\alpha) - m_2(\sin\beta + \mu\cos\beta)]}{(m_1 + m_2)}$, where α and β are angles of inclination for plane on which m_1 and m_2 are place respectively

6. (a) (i) State the laws of solid friction

(ii) With the aid of a well labeled diagram describe an experiment to determine th e coefficient of kinetic of kinetic friction between the two surfaces.

(b) A body slides down a rough plane at 300 to the horizontal. If the coefficient of kinetic friction between the body and the plane is 0.4. Find the velocity after the body has travelled 6m along the plane.[4.2521ms^{-1}]

7. (a)(i) State the laws of friction between solid surfaces

(ii) Explain the origin of friction force between two solid surfaces in contact.

(iii) Describe an experiment to measure the coefficient of kinetic friction between two solid surfaces

(b) (i) A car of mass 100kg moves along a straight surface with a speed of 20ms-1. When brakes are applied steadily, the car comes to rest after travelling 50m. Calculate the coefficient of friction between the surface and the tyres. [μ =0.4077]

(ii) State the energy changes which occur from the time the brakes are applied to the time to the time the car comes to rest.

[kinetic energy →heat → sound energy]

(c)(i) State the disadvantages of friction

[Wears tyres, produces unnecessary noise]

(ii) Give one method of reducing friction between solid surface.[by lubrication]

8. A block of mass 6.0kg is projected with a velocity of 12ms^{-1} up a rough plane inclined at 45^0 to the horizontal. It travels 5.0m up the plane. Find the frictional force. [44.8N]

9. (a) state the laws of friction

(b) A block of mass 5.0kg resting on the floor is given horizontal velocity of 5.0ms^{-1} and comes to rest in a distance of 7.0m. Find the coefficient of kinetic friction between the block and the floor.

(c) A car of mass 1500kg rolls from rest down a road inclined to the horizontal at an angle of 35^0, through 50m. The car collides with another car of identical mass at the bottom of incline. If the two vehicles interlock on collision and coefficient of kinetic friction is 0.20, find the common velocity of the vehicles [v = 10.024ms^{-1}]

These are properties of material under the action of force

(a) Strength: this is the ability of a material to resist breaking when a force is applied. E.g. metals
(b) Stiffness: this is the ability of a material to resist change in shape or size when a force is applied to it e.g. glass, dry wood.
(c) Elasticity is the ability of a material to regain its original shape and size after its deforming force has been removed. E.g. rubber, nullified spring.
(d) Plasticity: This is the ability of a material to remain permanently deformed when a deforming force has been removed e.g. plasticine, wet clay
(e) Ductility: this is the ability of a material to be changed into various shapes without breaking or is the ability of a material to be deformed without breaking e.g. metals

    Ductile materials undergo both elastic and plastic deformation.

(f) Brittleness: this is the ability of a material to break easily or suddenly when a force is applied to it e.g. dry clay, glass, chalk. Brittle materials bend very little and break. Therefore they undergo only elastic deformation and their elastic region is very small.
(g) Hardness/toughness: this is the ability of a material to resist wearing e.g. rubber and metals

Hooke's law

This states that the extension produced in a wire is directly proportional to the force applied provided the elastic limit is not exceeded.

$F \propto e$

F = ke

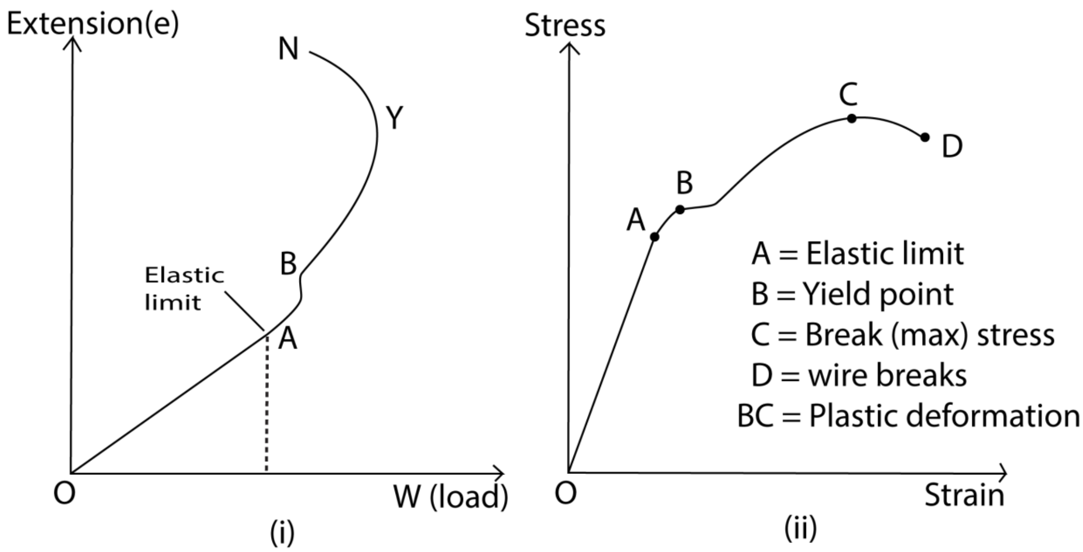
k= $\frac{F}{e}$ where k = force constant

Units of k

k = $\frac{MLT^{-2}}{L} = MT^{-2}$ or ks^{-1} or Nm^{-1}

Force constant, k, is the force required to change length of material by 1m.

Graph of force against extension



Extension versus Load

Description of section of stress- strain curve for ductile material above

Elastic limit:
This is the stress/load/point beyond which a material stops undergoing elastic deformation.
Yield point
Is the stress/load/point beyond which a material stops undergoing plastic deformation
NB. At yield point, there is a sudden increase in extension even though a small force is used.

Region OA
Extension is directly proportional to the force applied (stress is directly proportional to strain).
The material regains its original length and shape when a force/stress is removed. Extension produced is due to the molecules being displaced from their equilibrium position.

Region AB
Material regains its original shape when the force /stress are removed.

Stress is not proportional to strain or extension is not directly proportional to applied force therefore the material does not obey Hooke's law.

Region BC
the material does not fully regain its original shape and length when the force/stress is removed. Therefore Hooke's law is not obeyed.

The extension produced is due to the atoms of molecules being pulled apart breaking the bond between them. When the stretching forces are removed, these bonds are never recovered therefore, the material does not fully regain its original length.

Region CD

The material regains permanently stretched or deformed when the force/stress is removed. The bonds between the molecules are broken completely.

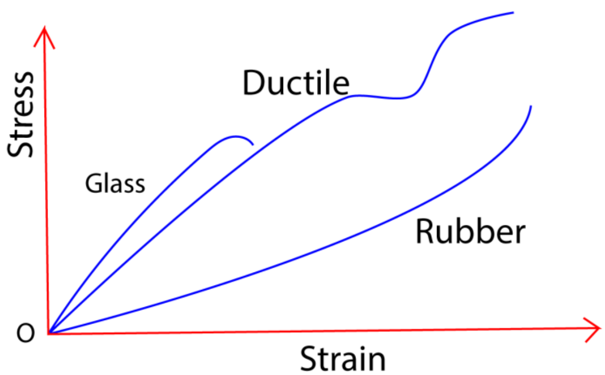
Region DE
The wire breaks in this region with any further increase in force or stress.

At breaking point the wire thins out, become hot. i.e. heat is given out.

Elastic deformation
During elastic deformation, molecular separation increases. This increases elastic potential energy when the force/stress is removed; the molecules regain their equilibrium position. Initial elastic potential energy is regained and stability is restored.

Plastic deformation
During plastic deformation, molecular separation increases leading to gain in elastic potential energy. Some of this energy is then lost in form of heat. When the force/stress is removed, the lost heat is never regained and therefore stability is never restored.

Stress-strain curve for non-ductile material



Glass: has he smallest elastic region and no plastic deformation regions. Glass is brittle due to small cracks on its surface. Any concentration of tensile stress/force on any of these cracks makes the glass break.

Rubber
Stretches easily without breaking and has a greatest range of elasticity. It it does not undergo plastic deformation

Unstretched rubber consist of coiled molecules when a tensile force is applied, they uncoil, become straight and hard. Any further increase in tensile force makes the rubber to break.

Work – hardening
When a metal is repeatedly deformed, it becomes brittle and its resistance to plastic deformation increases. This is called work- hardening of a metal.

Stress
This is the force acting on an area of $1m^2$ of a material or it is force per unit area

$$Stress = \frac{Force}{Area}$$

$$[s] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Units = $kgm^{-1}s^{-2}$, Nm^{-2} or Pascal (Pa)

Strain
This is the change in length per unit original length or is the change in length per 1m of original length

Strain = $\frac{Extension}{original\ length} = \frac{e}{L}$ (no units)

Young's modulus of elasticity
Young's modulus, E, is the ratio of tensile stress to tensile strain

E = $\frac{Tensile\ stress}{Tensile\ strain}$

[Y] = $\frac{MLT^{-2}}{L^2} = ML^{-1}T^{-1}$

Units = $kgm^{-1}s^{-2}$, Nm^{-2} or Pascal (Pa)

Or

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$

Then

F = $\frac{EeA}{L}$

Example 1
A mass of 2kg attached to the end of a wire 2m and diameter 0.64mm causes an extension of 0. 6mm. Find the Young's modulus.

F = 2 x 9.81 = 19.62N, A = $2\pi r^2 = \pi\ x\ (0.32\ x\ 10^{-3})^2 =$ 3.22 x $10^{-7}m^2$

$Stress = \frac{19.62}{3.22x\ 10^{-5}}$ = 6.1 x 10^7Nm^{-2}

And

$Strain = \frac{e}{L} = \frac{0.6\ x\ 10^{-3}}{2}$ = 3 x 10^{-4}

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{6.1\ x\ 10^7}{3\ x\ 10^{-4}}$ = 2.03 x 10^{11} Nm^{-2}

Alternatively

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$

E = $\frac{2\ x\ 9.81\ x\ 2}{\pi\ 0.32\ x\ 10^{-6}\ x\ 0.3\ x\ 0.3\ x\ 10^{-3}}$ = 2.03 x 10^{11}Nm^{-2}

It should be notes that Young's modulus, E, is calculated from ratio stress/ strain with the elastic limit of the material.

Example 2

Find the maximum load in kg in which may be placed on a steel wire of diameter 0.10cm if the permitted strain must not exceed 0.001 and Young's modulus for steel is 2.0 x 10^{11}Nm$^{-2.}$

E = $\frac{Tensile\ stress}{Tensile\ strain}$

Maximum stress = maximum strain x Young's modulus

= 0.001 x 2 x 10^{11} = 2 x 10^{8}Nm^{-2}

Area of cross-section in m^2 = $\frac{\pi d^2}{4} = \frac{\pi\ x\ (0.1\ x\ 10^{-2})^2}{4}$ = 7.85 x 10^{-7}m^2

Stress = $\frac{Force}{Area}$

Force = stress x area = 2 x 10^8 x 7.85 x 10^{-7} = 157N

Mass = $\frac{F}{g} = \frac{157}{9.81} = 16kg$

Force in bar due to contraction or expansion

When a bar is heated, and then prevented from contracting as it cools, a considerable force is exerted at the end of the bar.

For a bar which is L m having Young's modulus, E, a cross-sectional area, A, a linear expansivity of magnitude, α, and decrease in temperature, t^0 C; the decrease length e if were free to contract will be = αLt

E = $\frac{Tensile\ stress}{Tensile\ strain}$

$= \frac{F}{A} \div \frac{e}{L}$

$= \frac{FL}{eA}$

$F = \frac{EA\alpha Lt}{L}$

$= EA\alpha t$

Example 3

A steel rod of cross section area $2.0cm^2$ is heated to 100^0C, and then prevented from contracting when it is cooled to 10^0C. the linear expansivity of steel = $12 \times 10^{-6}K^{-1}$ and Young's modulus, E = $2.0 \times 10^{11}Nm^{-2}$. Find the force exerted.

Solution

A = $2cm^2 = 2 \times 10^{-4}\ m^2$, t = 100 – 10 = 90^0C

$F = EA\alpha t$

= $2 \times 10^{11} \times 2 \times 10^{-4} \times 12 \times 10^{-6} \times 90$

= 43200N

Energy stored in a stretching wire

When a wire is stretched by an amount, e, by applying a force, F without exceeding elastic limit. The average force = $\frac{(0+F)}{2} = \frac{1}{2}F$

Work done/ work stored in a wire = force x distance = $\frac{1}{2}F.e$

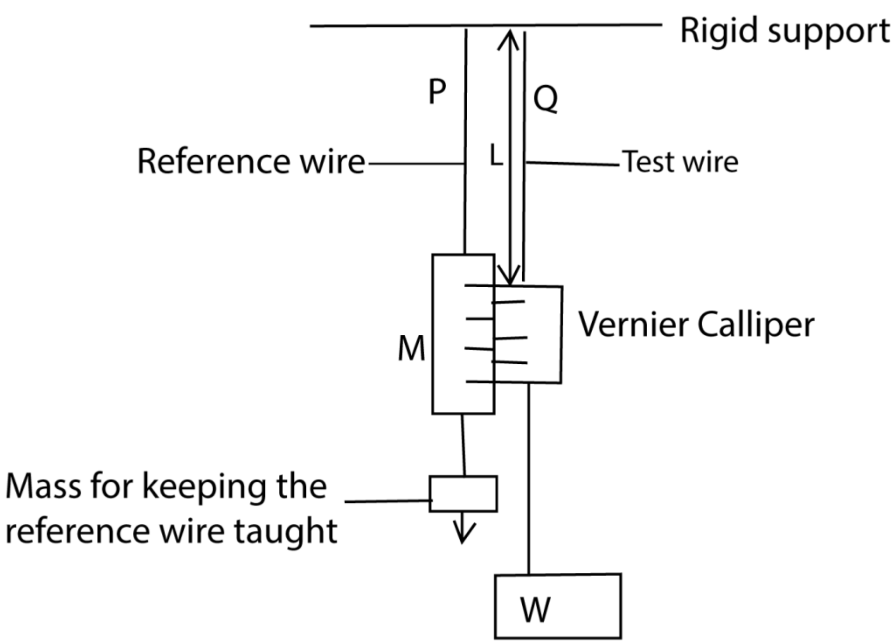
Work done per unit volume of a wire

The volume of the wire = AL

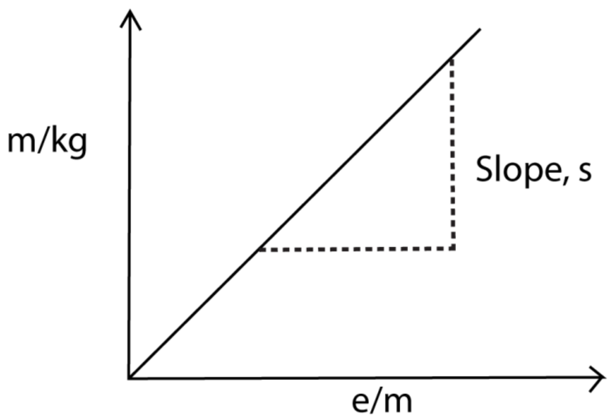
Energy per unit volume = $\frac{1}{2}F.e \div AL = \frac{1}{2}\ x\ \frac{F}{A}\ x\ \frac{e}{L}$

Energy stored per unit volume = $\frac{1}{2}\ x\ stres\ x\ strain$

Experiment to determine Young's Modulus for a metal wire



(i) Two thin, long wires of the same material and length P and Q are suspended from a rigid support.
(ii) P carries a scale M in mm and its straightened by attaching a weight at its end.
(iii) Q carries a vernier scale which is alongside scale M
(iv) Various loads are added to the test wire and corresponding extensions caused are read off from a vernier scale.
(v) The diameter (2r) of the wire is obtained by a micrometer screw gauge, and the cross section area of the wire A = $4\pi r^2$
(vi) A graph of mass (m) of the load against extension e is plotted



Young's modulus, Y = $\frac{gsL}{A}$

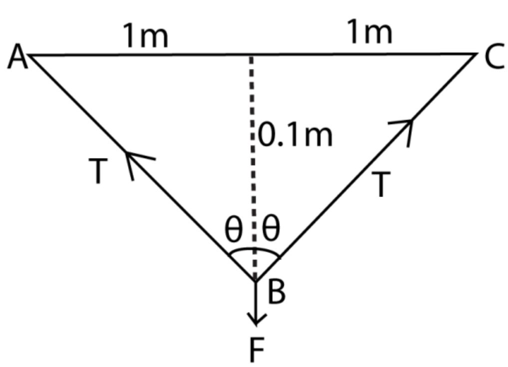
Precaution in the experiement above

1. After each reading, the load is removed to check that the wire returns to its original length, to ensure that elastic limit is not exceeded.
2. Long wires are used to achieve measurable expansion
3. Thin wires are used to produce high tensile stress
4. Identical wires are used to eliminate error of expansion or contraction due to changes in temperature.

Example 4

A metal wire of diameter 2.0 x 10^{-4}m and length 2m is fixed horizontally between two points 2m apart. Young's modulus for the wire is 2 x 10^{11}Nm^{-2}.

(i) What force should be applied at the midpoint of the wire to depress it by 0.1m
(ii) Find the work done



$\cos\theta = \frac{0.1}{AB}$ but $AB = \sqrt{(1^2 + 0.1^2)}$ = 1.005m

$\cos\theta = \frac{0.1}{1.005}$

Length ABC = 2AB = 2 x 1.005 = 2.01
Extension, e = 2.01 – 2 = 0.01m

T = $\frac{YAe}{l}$ and A = $A = \pi r^2 = \frac{\pi d^2}{4}$
Resolving vertically
F = 2Tcosθ
F = $\frac{2YAecos\,\theta}{L} = \frac{2Y\pi d^2 ecos\theta}{4L}$

F = $\frac{2\ x\ 2\ x\ 10^{11} x\ \pi\ x\ (2\ x\ 10^{-4})^2\ x\ 0.01\ x\ 0.1}{1\ x\ 4\ x\ 1.005} = 12.5N$

(iii) Work done = $\frac{1}{2}Fe = \frac{1}{2}\ x\ 12.5\ x\ 0.01 = 0.0625J$

Example 5

A uniform bar of length 1.0m and diameter 2.0cm is fixed between two rigid supports at 25^0C. If the temperature of the rod is raised to 75^0C.
Find
(i) The force exerted on the supports.
(ii) The energy stored in the rod at 75^0C.
(Young's modulus for the metal = 2.0 x 10^{11}Pa,
coefficient of linear expansion = 1.0 x $10^{-5}K^{-1}$)

Solution

(i) F = YAαΔθ
= 2.0 x 10^{11} x $(\pi\ x\ 0.01^2)$ x 1.0 x 10^{-5} (75 – 25)
= 31400N
(ii) Energy stored = $\frac{1}{2}Fe$ but e = αLΔθ
= $\frac{1}{2}\ x\ 31400\ x\ 1\ (75 - 25) = 7.85$J

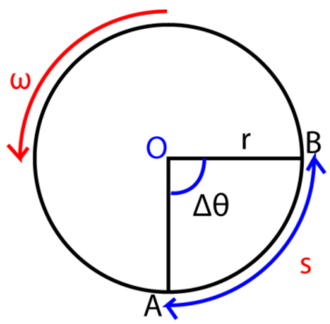
Exercise

1. A uniform wire of upstretched length 2.49m is attached to two points A and B which are two meters apart and in the same horizontal line. When 5kg mass is attarched to the midpoint, c, of the wire, the equilibrium point of c is 0.75m between the line AB. (Young's modulus for the wire =2.0 x 10^{11}Pa)
Find
(i) Strain and stress in the wire [strain = 0.00402, stress = 8.04 x 10^8Pa]
(ii) Energy stored in the wire [Ans. 2.02 x 10^3J]

2. A thin steel wire initially 1.5m long and diameter 0.5mm is suspended from a rigid support. Calculate
   (i) extension (Ans. 3.53×10^{-3}m)
   (ii) the energy stored in the wire, when a mass of 3kg is attached to the lower end. [5.19×10^{-2}J]
   (Young's modulus = $2.0 \times 10^{11}Nm^{-2}$]
3. Two thin wires, one of steel and the other of bronze each 1.5mg and the diameter 0.2cm are joined end to end to form a composite wire of 3m. What tension will produce a total extension of 0.064cm? (Young's modulus for steel = 2.0×10^{11}Pa, Young's modulus of bronze = 1.2×10^{11}Pa) {Ans. 1009N]

Circular motion

This is the motion of an object moving in a circular path with a uniform speed around a fixed point O. consider a body moving from A to B in a small time Δt such that the radius r, sweeps through a small angle Δθ in radians



Distance, AB = s = rΔθ

But speed, v = $\frac{distance}{time} = r\frac{\Delta\theta}{\Delta t}$

As Δt → 0, $\frac{\Delta\theta}{\Delta t} \rightarrow \frac{d\theta}{dt}$

v = $r\frac{d\theta}{dt}$

Since, $\frac{d\theta}{dt}$ = ω

v = rω

Terminology in circular motion

Angular velocity, ω

This is the rate of the angle for an object moving in a circular path about in a circular path about the center. S.I units are rads^{-1}.

Period, T

Time taken to make one complete revolution

Time = $\frac{Distance}{speed}$

= $\frac{2\pi r}{v}$

= $\frac{2\pi r}{r\omega}$

= $\frac{2\pi}{\omega}$

Frequency, f
This is the number of revolutions made in one second. The S.I units are hertz (Hz)

$F = \frac{1}{T}$

$= \frac{\omega}{2\pi}$

Centripetal acceleration (a)
This is the rate of change of velocity for a body moving in a circular path and it is directed towards the center of that circular path.

$a = \frac{v^2}{r} = r\omega^2$

Derivation of a = $\frac{v^2}{r}$ *or* $r\omega^2$

Consider an object moving with a constant speed, v, round a circle of radius, r.

In figure (i) below; at A, its velocity v_A is in direction of the tangent AC, a short time dt later at B, its velocity v_B is in the direction of tangent BD

Since their directions are different, the velocity v_B is different from the velocity v_A although their magnitude are both equal to v.

Thus a velocity change or acceleration has occurred from A to B

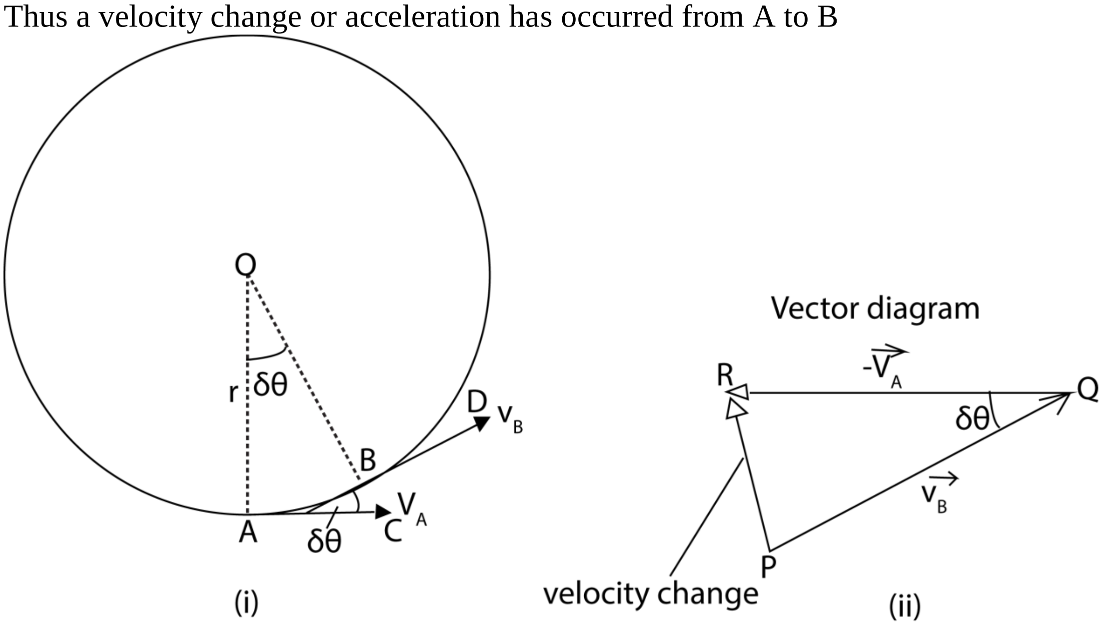


Fig. 2: Acceleration in circle

The velocity change from a to B = $v_B - v_A$ or $v_B + (-v_A)$.

In figure 2(ii) above, PQ represents v_B in magnitude (v) and direction BD; QR represents $-v_A$ in magnitude (v) and direction (CA).

Velocity change = $v_B + (-v_A)$ = PR

When δt is small, the angle AOB or δθ is small;
Also angle PQR equal to δθ is small

PR or acceleration then points toward O, the center of the circle.

a = $\frac{velocity\ change}{time} = \frac{PR}{\delta t} = \frac{v\delta\theta}{\delta t}$

but $\frac{\delta\theta}{\delta t}$ =ω and v = rω

a = rω x ω = $r\omega^2$

thus an object moving in a circle of radius r with a constant speed v has a constant acceleration towards the center equal to $\frac{v^2}{r} = r\omega^2$

Centripetal force

This is the force which keeps the body moving in a circular path and it is directed towards the center of the circular path.

F = ma = m$\frac{v^2}{r} = mr\omega^2$

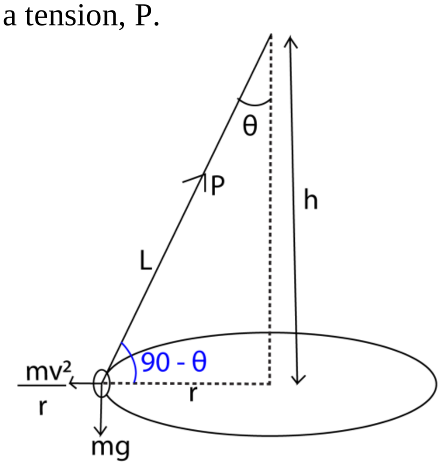
Centrifugal force

This is the force acting on a body moving in a circular path.

Examples of circular motion

1. A conical circular pendulum

Consider a mass , m, attached to a string of length L moving around a horizontal circle of radius, R, at a constant speed, v, and the string make an angle θ with the vertical and have a tension, P.



Resolving vertically

Pcosθ = mg ……………………………………………….. (i)

Resolving horizontally

Psinθ = m$\frac{v^2}{r}$ ……………………………………………… (ii)

Eqn (i) and Eqn (ii)

tan θ =$\frac{v^2}{rg}$

From the diagram

sin θ = $\frac{r}{L}$, r = Lsinθ

cos θ = $\frac{h}{L}$, r = Lcos θ

Pcos θ = mg

⇨ P . $\frac{h}{L}$ = mg

P = $\frac{mgL}{h}$ …………………………….. (iii)

Psinθ = m$\frac{v^2}{r}$

P . $\frac{r}{L}$ = m$\frac{v^2}{r}$

P = $\frac{mv^2L}{r^2}$

But v = rω

P = $\frac{m(r\omega)^2L}{r^2}$ = mLω^2 …………………….. (iv)

Eqn (iii) and (iv)

mLω^2 =$\frac{mgL}{h}$

$\omega^2 = \frac{g}{h}$

From T = $\frac{2\pi}{\omega}$

ω =$\frac{2\omega}{T}$

$\frac{g}{h} = \left[\frac{2\pi}{T}\right]^2 = \frac{4\pi^2}{T^2}$

$T^2 = \frac{4\pi^2 h}{g}$

T = $\sqrt{\frac{2\pi^2 h}{g}}$ = 2π$\sqrt{\frac{h}{g}}$

Example 1

A mass of 0.2kg is whirled in a horizontal circle of radius 0.5m by a string inclined at 300 to the vertical. Find

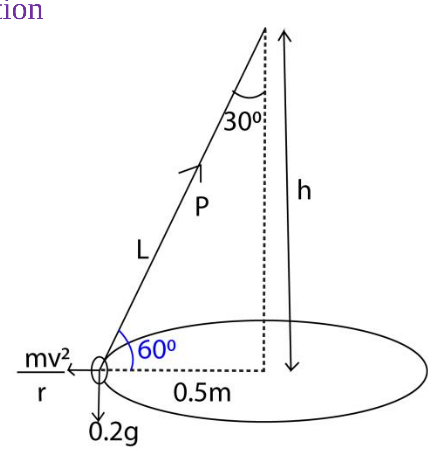
(i) The tension in the string

(ii) The speed of the mass in the horizontal plane

(iii) The length of the string

(iv) The angular speed

Solution



(i) Resolving vertically

Pcos θ = mg

$P = \frac{mg}{\cos\theta} = \frac{0.2 \ x \ 9.81}{\cos 30} = 2.2655N$

Tension P = 2.2655N

(ii) Resolving vertically

$P\sin\theta = m\frac{v^2}{r}$

$v^2 = \frac{rP\sin\theta}{m}$

$v = \sqrt{\frac{rP\sin\theta}{m}} = \sqrt{\frac{(0.5 \ x \ 2.2655 \ x \ \sin 30^0}{0.2}} = 1.6828ms^{-1}$

(iii) From $\sin\theta = \frac{r}{L}$

$L = \frac{0.5}{\sin 30} = 1m$

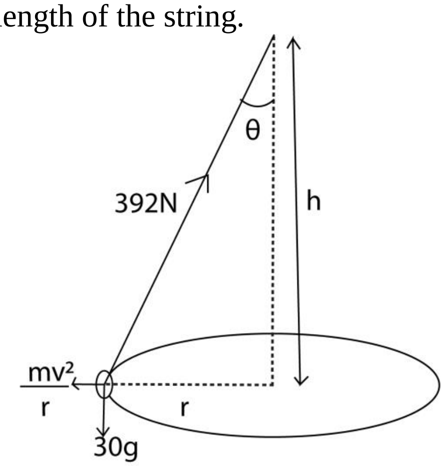
(iv) From $\omega = \frac{v}{r} = \frac{1.6828}{0.5} = 3.265rads^{-1}$

Example 2

A 30kg body is swirled in a horizontal circle as a conical pendulu by means of inelastic string that has a breaking strength of 392N. when the speed of the body is $8ms^{-1}$, the string broke. Calculate

(i) The angle the string made at that instant.

(ii) The length of the string.



(i) From $P\cos\theta = mg$

$\cos\theta = \frac{30 \ x \ 9.81}{392}$

$\theta = 41.34$

(ii) From $P\sin\theta = m\frac{v^2}{r}$

$r = \frac{30 \ x \ 8^2}{392 \sin 41.34^0} = 7.42$

But $\sin\theta = \frac{r}{L}$

$L = \frac{7.42}{\sin 41.34} = 11.23m$

Example 3

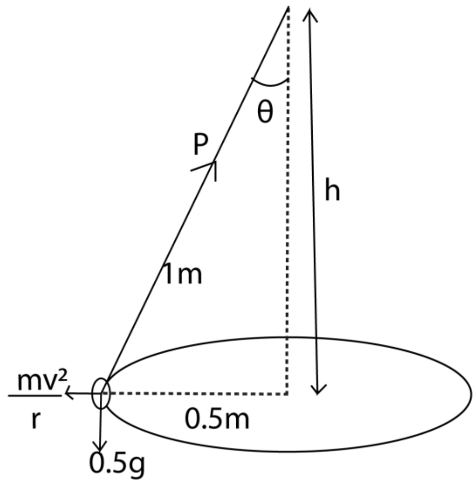
A steel ball of mass 0.5kg is suspeneded from a light inelastic string of length 1m, the ball is whirled in horizontal cirle of radis 0.5m. find

(i) Centripetal force and tension in the string

(ii) The angular speed of the ball

(iii) The angle between the string and the radius of the circle is thetension in string is 10N

Solution



(i) From $\sin\theta = \frac{r}{L}$

$\theta = \sin^{-1}\frac{0.5}{1} = 30^0$

From $P\cos\theta = mg$

Tension, $P = \frac{mg}{\cos\theta} = \frac{0.5\ 9.81}{\cos 30} = 5.664N$

But, $P\sin\theta = F$

$F = 5.664 \sin 30 = 2.832N$

(ii) $\tan\theta = \frac{v^2}{rg}$

$v = \sqrt{rg\tan\theta} = \sqrt{[0.5\ x\ 9.81\ x \tan 30^0]} = 1.683$

But, $\omega = \frac{v}{r} = \frac{1.683}{0.5} = 3.366 rads^{-1}$

(iii) From $P\cos\theta = mg$

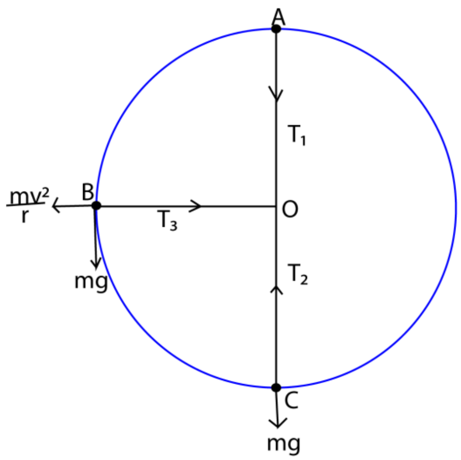
$\cos\theta = \frac{0.5x\ 9.81}{10}$

$\theta = 60.60$

the required angle = 90 – 60.6 = 29.4^0

Motion in verticle circle

Consider a body of mass, m, attached to a string of length, r, and whirled in a vertical circle at constant velecity, v.



At A, $m\frac{v^2}{r} = T_1 + mg$

$T_1 = m\frac{v^2}{r} - mg$

At B, $T_3 = m\frac{v^2}{r}$

At C, $T_4 = m\frac{v^2}{r} + mg$

From the above expressions, tension in the string is minimum aat the top of the circle and maximum at the bottom of the circle. So the string is most likely to break when the body is at the bottom of the circle.

Example 4

A mass of 0.4kg is rotated by a string at a constant speed, v, in a vertical circle of radius 1m. If the minimum tension in a string is 3N. calculate

(i) The velocity

(ii) The maximum tension

(iii) Tension when the string is just horizontal

Solution

Minimum tension, $T_1 = m\frac{v^2}{r} - mg$

$3 = \frac{0.4v^2}{1} - 0.4\,x\,9.81$

$v = 4.16ms^{-1}$

Maximum tension $T_3 = m\frac{v^2}{r} + mg$

$= \frac{0.4\,x\,4.16^2}{1} + 0.4\,x\,9.81$

$= 10.85N$

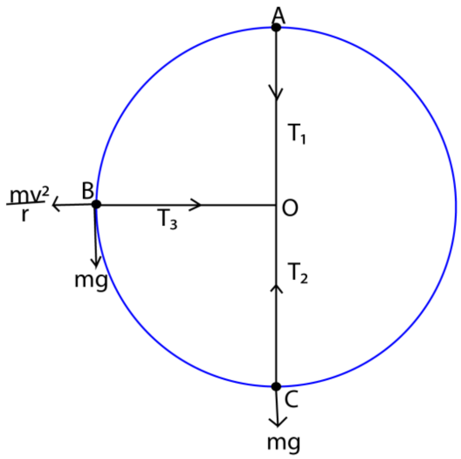
Tension when the string is horizontal $T = m\frac{v^2}{r} = \frac{0.4\,x\,4.16^2}{1} = 6.92N$

Example 5

A particle of mass 5kg describes a complete vertical circle at the end of a light inextensible string of length 2m. given that the speed of the particle is $5ms^{-1}$ at the highest point. Find

(i) Speed at the lowest point
(ii) Tension in the string when it is horizontal
(iii) Magnitude of centripetal acceleration when the string is horizontal

Solution



(i) Mechanical energy at A = m$\frac{v^2}{r}$ + mg

r = 2m, m = 5kg, v = $5ms^{-1}$

Mechanical energy at A = $\frac{1}{2} x\ 5\ x\ 5^2 + 5\ x\ 9.81\ x\ 4$ = 258.7J

Mechanical energy at C = m$\frac{v^2}{r}$ + mg = $\frac{1}{2} x\ 5\ x\ v^2 + 5\ x\ 9.81 x\ 0$ = $2.5v^2$ J

But from the principle of conservation of energy mechanical energy

Mechanism energy at A = mechanical energy a t C

$258.7 = 2.5v^2$

v = $10.2ms^{-1}$.

(ii) Mechanical advantage at B, = m$\frac{v^2}{r} + mgh$ = 5 x $\frac{v^2}{2}$ + 5 x 9.81 x 2 = 258.7

v = 8ms

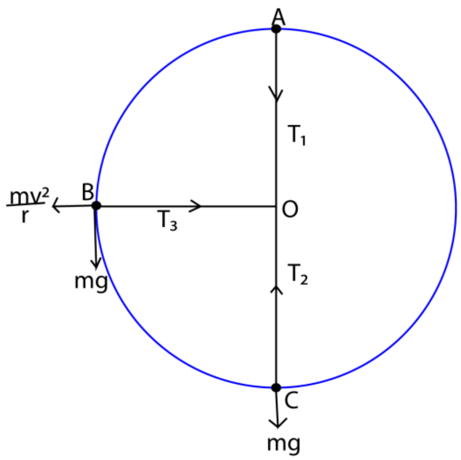
tension at B = m$\frac{v^2}{r}$ = $5\ x\ \frac{8^2}{2}$ =160N

(iii) From a = $\frac{v^2}{r} = \frac{8^2}{2}$ =$32ms^{-1}$

Example 6

A particle of mass m describes a complete vertical inextensible string of length, r, given that the speed at the lowest point is twice the speed at highest point. Show

(i) The speed of the particle at the lowest point is = v= $4\sqrt{\frac{gr}{3}}$.
(ii) The tension in the string when the particle is at the highest point, T = $\frac{mg}{3}$



(i) Mechanical energy at A = mechanical energy at B = m$\frac{v^2}{r}$ + mg

Let the speed at A = u

The speed at C = 2u

⇨ $m\frac{u^2}{2} + mg.2r = m\frac{(2u)^2}{2} + mg\ x\ 0$

$u^2 + 4gr = 4vu^2$

$3u^2 = 4gr$

$u = 2\sqrt{\frac{gr}{3}}$.

At the lowest point velocity = 2u = $4\sqrt{\frac{gr}{3}}$.

(ii) Tension at the highest point = $m\frac{v^2}{r} - mg = \frac{4mgr}{3r} - mg = \frac{4mg-3mg}{3} = \frac{mg}{3}$

Exercise

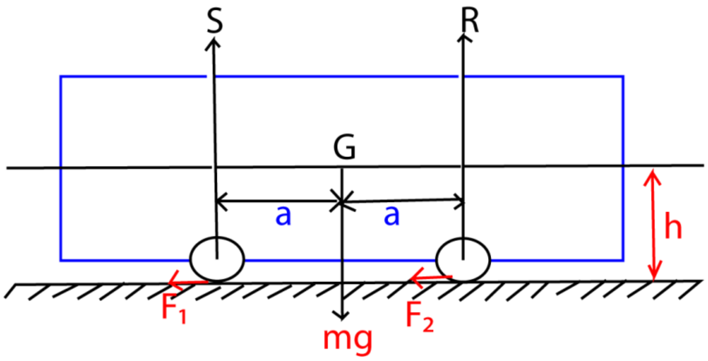
1. An object of mass 0.kg on the end of a string is whirled around in a horizontal circle of radius 2m, with a constant speed of 10ms^{-1}. Find its angular velocity and the tension in the string [Ans. ω = 5rads^{-1}, T = 25.5N)
2. A small ball of mass 0.1kg is suspended by an inextensible string of length 0.5m and is caused to rotate in a horizontal circle of radius 0.4m. find
   (i) The tension in the string [Ans. 1.3N]
   (ii) The period of rotation [ans. 1.2s]
3. A pendulum bob of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. the bob moves in a horizontal circle with the string inclined at 30^0 to the vertical. Calculate
   (i) Tension in the string [Ans. 2.27N]
   (ii) The period of motion [Ans. 2.02s]
4. The period of oscillation of a conical pendulum is 2.0s. if the string makes an angle 60^0 to the vertical at the point of suspension, calculate the
   (i) vertical height of the point of suspension above the circle [h = 0.994m]
   (ii) length of the string [L= 1.99m]
   (iii) velocity of mass attached to the string. [v = 5.41ms^{-1}]

Application of circular motion

(i) motion of the car around a circular level track (unbanked track)

(a) Overturning and toppling

Consider a car negotiating a bond on a level track. For a circular motion, the friction F1 and F2 provide the centripetal force.



Let 2a = the distance between the tyres,
h = height of the center of gravity of the car above the ground.
mg = weight of the car.
S and R = normal reaction on the tyres from the ground

Then,

$F_1 + F_2 = m\frac{mv^2}{r}$ ……………………………………………….(i)

Since the car does not move off the road, the sum upward force is equal to the sum of downward forces

R + S = mg ……………………………………………… (ii)

Taking moments about G

$Sa + F_1h + F2h = Ra$

$h(F_1 + F_2) = a(R - S)$

Substituting F1+ F2 from Eqn (i)

$m\frac{mv^2}{r}h = a(R\text{-}S)$

$(R\text{-} S) = m\frac{mv^2}{r}\ x\ \frac{h}{a}$……………………………………….. (iii)

Eqn. (ii) + Eqn. (iii)

$2R = mg + m\frac{mv^2}{r}\ x\ \frac{h}{a}$

$R = \frac{m}{2}\left(g + \frac{v^2}{ra}h\right)$

Eqn. (ii) – Eqn. (iii)

$2S = mg\ \text{-}\ m\frac{mv^2}{r}\ x\ \frac{h}{a}$

$S = \frac{m}{2}\left(g - \frac{v^2}{ra}h\right)$

If S = 0 , then the car is just about to topple/overturn

$0 = \frac{m}{2}\left(g - \frac{v^2}{ra}h\right)$

Either $\frac{m}{2} = 0$

or $\left(g - \frac{v^2}{ra}h\right) = 0$

$g = \frac{v^2}{ra}h$

But v = $\sqrt{\frac{gra}{h}}$

A car topples when v > $\sqrt{\frac{gra}{h}}$

Condition for overturning/toppling

(i) The center of gravity of the car is high

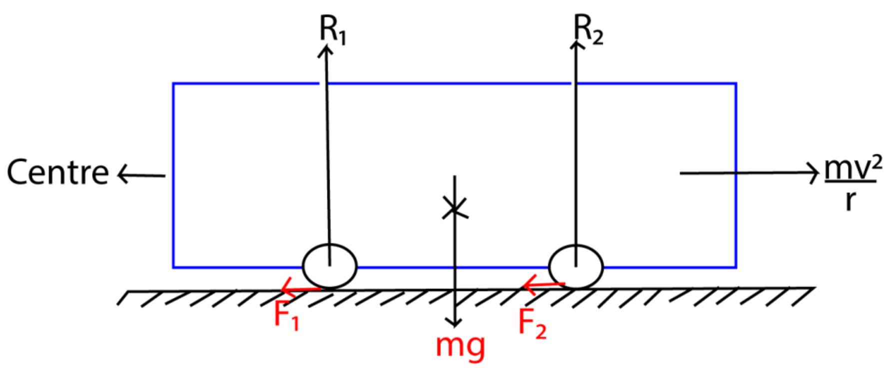
(ii) If the bend is sharp

(iii) When the distance between the tyres is small

(b) Sliding/ slipping/skidding

A vehicle skid when the available centripetal force is not enough to balance the centrifugal force. The vehicle fails to negotiate the curve and goes off the track outwards.

Consider a vehicle of mass, m, taking unbanked curve of radius, r, at speed, v.



$F_1 + F_2 = m\frac{v^2}{r}$

But $F_1 = \mu R_1$ and $F_2 = \mu R_2$

$\mu R_1 + \mu R_2 = m\frac{v^2}{r}$

$\mu (R_1 + R_2) = m\frac{v^2}{r}$

$\mu mg = m\frac{v^2}{r}$

$\mu g = \frac{v^2}{r}$

$v^2 = \mu gr$

v = $\sqrt{\mu gr}$ (this the maximum velocity for which the vehicle does not skid/slip

A vehicle skid/ slips when v >$\sqrt{\mu gr}$

Conditions that lead to skidding

(i) Sharp bends

(ii) Slippery roads

(iii) Very high speed

Example 6

A car goes around unbanked curve at $15ms^{-1}$, the radius of the curve is 60m. Find the least coefficient of kinetic friction that will allow the car to negotiate the curve without skidding.

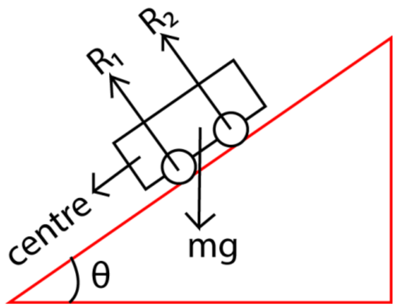
Solution

μ≥ $\frac{v^2}{rg} = \frac{15^2}{(60\ x\ 9.81)} = 0.38$

Motion of a car round a banked inclined track

Angle of banking

Consider car negotiating a bend inclined at an angle θ to the horizontal. It is assumed that there is no tendency to slip at the wheels, therefore no frictional forces.



Resolving horizontally

$R_1\sin\theta + R_2\sin\theta = m\frac{v^2}{r}$ ………………………………(i)

Resolving vertically

$R_1\cos\theta + R_2\cos\theta = mg$ ………………………………(ii)

Eqn (i) ÷ Eqn (ii)

$\frac{R_1\sin\theta + R_2 sin\theta}{R_1\cos\theta + R_2 cos\theta} = {m\frac{v^2}{r}}\Big/{mg}$

$\tan\theta = \frac{v^2}{rg}$

$v^2 = rgtan\ \theta$

$v = \sqrt{rgtan\ \theta}$

Example 7

(a) A bend on a level road form a circular arc of radius 54m. Find the highest speed at which the car can travel around the bend without slipping occurring. If the coefficient of friction between the car tyres and the road surface is 0.3

(b) A car of mass 400kg turns a corner at 40kmhr^{-1} without skidding but at 50kmh^{-1}, it skids off. If the corner forms an arc of radius 20m. Find the values between which the coefficient of friction between the wheels and the road surface lies.

Solution

(a) R = 54m, μ = 0.3

From v = $\sqrt{\mu gr}$

v = $\sqrt{(0.3\ x\ 9.81\ x\ 54}$ 12.60ms^{-1}

(b) Case (i)

Vmax = 40kmhr^{-1} = $\frac{40 \, x 1000}{3600} = \frac{100}{9} ms^{-1}$

r = 20m, m = 400kg

From v = $\sqrt{\mu g r}$

$$\mu = \frac{v^2}{rg} = \frac{\left[\frac{100}{9}\right]^2}{20 \, x \, 9.81} = 0.629$$

Case (ii)

Vmax = 50kmhr^{-1} = $\frac{50 \, x 1000}{3600} = \frac{125}{9} ms^{-1}$

From v = $\sqrt{\mu g r}$

$$\mu = \frac{v^2}{rg} = \frac{\left[\frac{125}{9}\right]^2}{20 \, x \, 9.81} = 0.983$$

Therefore, $0.629 \leq \mu \leq 0.983$

Example 8

A car of mass 300kg travels at 100kmhr^{-1} round unbanked curve of radius 200m.

(i) What is the minimum coefficient of sliding friction between the road and the tyres that will permit the car to negotiate the curve without skidding?

(ii) At what angle would the road be banked if there were no friction between the tyres and the road surface?

Solution

(i) From $\mu = \frac{v^2}{rg}$

v = 100kmhr^{-1} 10= $\frac{250}{9} ms^{-1}$

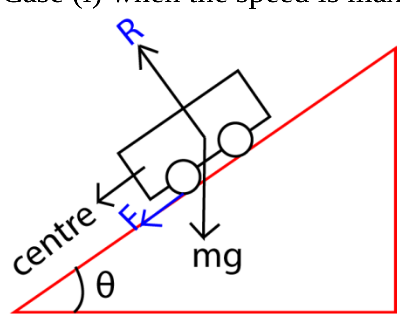
$$\mu = \frac{\left[\frac{250}{9}\right]^2}{200 \, x \, 9.81} = 0.393$$

From θ = tan^{-1} $\left[\frac{v^2}{rg}\right]$

$$\theta = tan^{-1}\left[\frac{\left(\frac{250}{9}\right)^2}{200 \, x \, 9.81}\right] = 21.5^0$$

(ii) Note that , all roads are banked and there is friction between the tyres and the road surface. The necessary centripetal force is provided by the horizontal component of normal reaction and friction force. There are two cases

Case (i) when the speed is maximum



Resolving horizontally

Rsinθ + Fcosθ = $m\frac{v^2}{r}$

But F = μR

Rsinθ + μRcosθ = $m\frac{v^2}{r}$ ……………………………(i)

Resolving vertically

Rcosθ -Fsinθ = mg

Rsinθ – μRsinθ = mg ……………………………(ii)

Eqn (i) ÷ Eqn (ii)

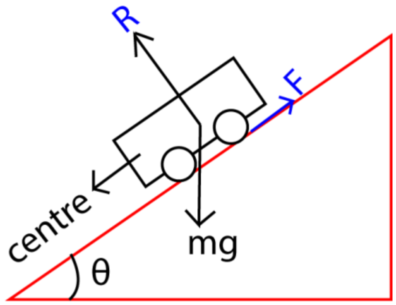
$$\frac{R\sin\theta + \mu R\cos\theta}{R\sin\theta - \mu R\sin\theta} = m\frac{v^2}{r}\Big/mg = \frac{v^2}{rg}$$

Dividing through by Rcosθ

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$$

$$v = \sqrt{\frac{rg\,(\tan\theta + \mu)}{1 - \mu\tan\theta)}}$$

Case (ii) When the speed is minimum



Resolving horizontally

Rsinθ - Fcosθ = $m\frac{v^2}{r}$

But F = μR

Rsinθ - μRcosθ = $m\frac{v^2}{r}$ ……………………………(i)

Resolving vertically

Rcosθ +Fsinθ = mg

Rsinθ + μRsinθ = mg ……………………………(ii)

Eqn (i) ÷ Eqn (ii)

$$\frac{R\sin\theta - \mu R\cos\theta}{R\sin\theta + \mu R\sin\theta} = m\frac{v^2}{r}\Big/mg = \frac{v^2}{rg}$$

Dividing through by Rcosθ

$$\frac{\tan\theta - \mu}{1 + \mu\tan\theta} = \frac{v^2}{rg}$$

$$v = \sqrt{\frac{rg\,(\tan\theta - \mu)}{1 + \mu\tan\theta)}}$$

Gravitation

Kepler's Law of Planetary Motion

1. Planets revolve in elliptical orbits having the sun at one focus

2. Each planet revolve in such a way that the imaginary line joining it to the sun sweeps out equal areas in equal times

3. The squares of the periods of revolution of the planets are proportional to the cubes of their mean distances from the sun

Newton's law of gravitation

The force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the squares of their distance apart.

Consider two bodies of mass M and m with a distance r apart

Force, $F \propto \frac{Mm}{r^2}$

$$F = G\frac{Mm}{r^2}$$

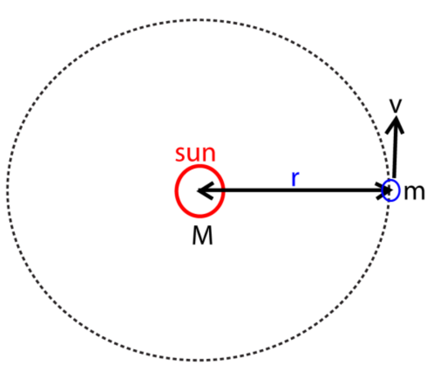
Where, G, is the universal gravitational constant.

$$G = \frac{Fr^2}{Mm}\ \mathrm{Nm^2kg^{-1}}\ \text{or}\ \mathrm{M^3kg^{-1}s^{-2}}$$

Numerical value of G = $6.67 \times 10^{-11}\mathrm{Nm^2kg^{-2}}$

Proof of Kepler's 3rd law

Consider a planet of mass m moves with speed *v* in a circle of radius r round the sun of mass M.



Gravitational attraction of the sun for the planet, F = $\frac{GMm}{r^2}$ ……. (i)

The centripetal force keeping the planet in orbit, F = $\frac{mv^2}{r}$ …….. (ii)
Equation (i) and (ii)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r} \text{ ………………………………………………….. (iii)}$$

But period of revolution, T = $\frac{2\pi}{\omega}$

$$\omega = \frac{v}{r}$$

$$T = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T} \text{ ………………………… (iv)}$$

Equation (iii) and (iv)

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

Since G, m, π are constant, then, $T^2 \propto r^3$ which verifies Kepler's 3rd law

Mass and density of the earth
For a body of mass, m, on the earth's surface, the force of gravity acting on it is given by F = mg

The earth of mass M and radius r is assumed to be spherical and uniform and thus its mass concentrated at its center. The force of attraction on the earth on the body is given by

F = $\frac{GMm}{r_e^2}$

$$\frac{GMm}{r_e^2} = mg$$

M = $\frac{gr_e^2}{G} = \frac{9.81 \, x \, (6.4 \, x \, 10^6)^2}{6.7 \, x \, 10^{-11}}$ since r_e = 6400km or 6.4 x 10^6m

= 6 x 10^{24}kg

Density

Since the earth is spherical

Density = $\frac{mass}{volume} = \frac{6 \, x \, 10^{24}}{\frac{4}{3}\pi r_e^3} = \frac{6 \, x \, 10^{24}}{\frac{4}{3} \, x \, 3.14 \, x \, (6.4 \, x \, 10^6)^2}$ = 5500kgm^3

Mass of the sun

Consider the earth of mass m revolving round the sun of Mass, M at a distance r from the sun

Centripetal force on the earth

$F = \frac{mv^2}{r}$

$v = \omega r$

$F = m\omega^2 r$

but $\omega = \frac{2\pi}{T}$

$F = \frac{4m\pi^2 r}{T^2}$ …………………………. (i)

Force of attraction between the earth and the sun is

$F = \frac{GMm}{r^2}$ …………………………. (ii)

Equation (i) and (ii)

$\frac{GMm}{r^2} = \frac{4m\pi^2 r}{T^2}$

$M = \frac{4\pi^2 r^3}{GT^2}$

T = 1year = 365 x 60 x 60 = 31, 536, 000s,

G = $6.67 \times 10^{-11} Nm^2kg$,

r = 150×10^6km= 1.5×10^{11}m

$M = \frac{4\pi^2(1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (31536000)^2} = 2 \times 10^{30}$kg

Assumptions

(i) The sun is stationary
(ii) The earth is spherical

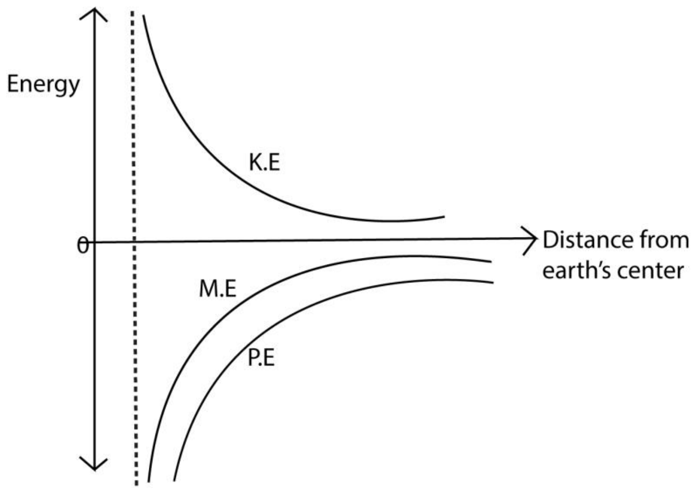
Example 1

State the effect of frictional forces on the motion of an earth satellite against distance from the earth's center.

Solution

The mechanical energy of a satellite decreases, potential energy decrease, kinetic energy increases. The velocity of the satellite increase while the the radius of the orbit decrease and satellite may burn if no precaution are taken.

A graph showing the variation of energy of a satellite against distance from the earth's center.

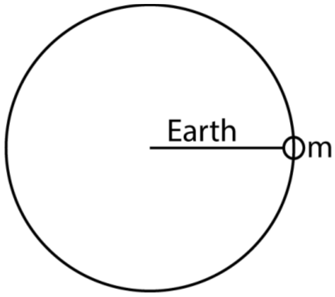


Escape velocity

This is the vertical velocity that should be given to a body at the surface of a planet so that it just escapes from gravitational attraction of the planet.

Derivation

Consider a body or a rocket of mass, m, fired from the earth's surface such that it escapes from the gravitational influence of the earth. However, it requires a certain velocity to escape.



At the surface

K.E = $\frac{1}{2}mv^2$

P.E = - $\frac{GM_e m}{r_e}$

When the rocket escapes is at infinity, Me = 0

By conservational of momentum

$\frac{1}{2}mv^2$ - $\frac{GM_e m}{r_e}$ = 0

$v^2 = \frac{2GM_e}{r_e}$

$v = \sqrt{\frac{2GM_e}{r_e}}$

This shows that the escape velocity is independent of the mass of the body.

Example 2

Explain why the moon has no atmospheres yet the sun has

Solution

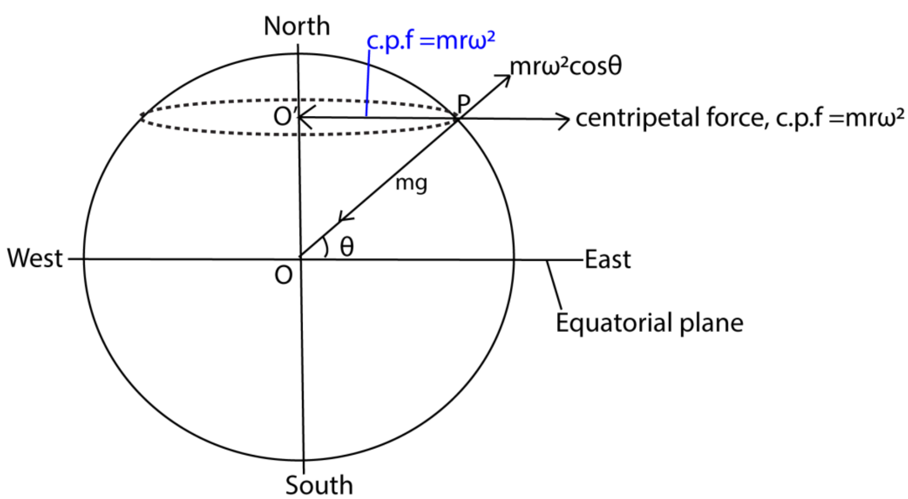
This is because the gravitational attraction at the moon is too weak to hold the gases due to its small mass and as a results, the gases escape leaving the moon without atmospheres.

Variation of acceleration due to gravity, g, with latitude

The acceleration due to gravity, g, varies over the earth because the earth is elliptical, with the polar radius, b = 6.357 x 10^6m and the equatorial radius, a = 6.378 x 10^7m, hence as one moves from the equator to the poles the distance of the point on the surface of the earth from the centre of the earth decreases. Hence the acceleration due to gravity or attraction of the body towards the center increases. (note that force of attraction due to gravity at a place inversely proportional to the distance of the point from the center of the earth squared)

The latitude of a point is the angle θ between the equatorial plane and the line joining that point to the center of the earth. Latitude of the equator is 0^0 and that of the pole is 90^0.

Consider a body of mass, m, at point P with latitude θ as shown on the surface of the earth and g_θ be the acceleration due to gravity at P.



OP = R = radius of the earth
o'P = r = distance of P from the axis of the Earth

Due to rotation motions of the earth about its axis P experiences a centripetal force given by $mr\omega^2$

Resolving the centripetal force into two rectangular component, its component along the radius of the earth = $mr\omega^2\cos\theta$

Also, the body is acted on by two forces; its weight acting towards the center of the earth and the component, $mr\omega^2\cos\theta$ acting radially outwards.

The difference in between the the two forces is the weight of the body at that point at that point

$mg_\theta = mg - mr\omega^2\cos\theta$ …………………. (i)

But $\cos\theta \frac{O\prime P}{OP} = \frac{r}{R}$

$\therefore r = R\cos\theta$ ………………………….. (ii)

Substituting equation (ii) in (i)

$mg_\theta = mg - m(R\cos\theta)\omega^2\cos\theta$

$g_\theta = g - R\omega 2\cos^2\theta$

This is the expression for acceleration due to gravity at point P on the earth's surface having latitude θ.

At equator $\theta = 0^0$, hence g_θ is minimum. At the pole $\theta = 90^0$, hence, g_θ is maximum.

Example 3

What is the weight of a body of mass 1000kg on the earth at

(a) Equator
(b) Pole
(c) Latitude of 300 (Radius of the earth, R = 6400km, acceleration due to gravity, g = $9.81ms^{-2}$)

Solution

Period of the earth, T = 24 x 60 x 60s

Angular velocity of earth, $\omega = \frac{2\pi}{T} = \frac{2\ x\ 3.14}{24\ x\ 60\ x\ 60} = 7.273\ x\ 10^{-5}\ rads^{-1}$

The weight of a body W_θ = latitude θis given by

$W_\theta = m(g - R\omega 2\cos^2\theta)$

(a) At the equator $\theta = 0^0$
$W_0 = 1000[9.81 - 6.4 \text{ x } 10^6(7.273 \text{ x}10^{-5})^2\cos^2 0]$
=9776.15N
(b) At the pole $\theta = 90_0$
$W_0 = 1000[9.81 - 6.4 \text{ x } 10^6(7.273 \text{ x}10^{-5})^2\cos^2 90]$
=9810N
(c) At the latitude where $\theta = 30_0$
$W_0 = 1000[9.81 - 6.4 \text{ x } 10^6(7.273 \text{ x}10^{-5})^2\cos^2 30]$
=9784.61N

Variation of acceleration to gravity due to depth

From mg = $\frac{GMm}{R^2}$

$$g = \frac{GM}{R^2} \text{ .... (i)}$$

where M is the mass of the earth.

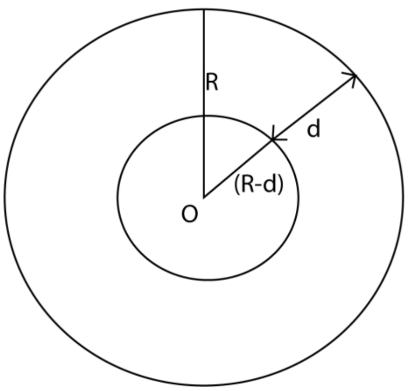
But M = volume x density, ρ, of the earth's material

$$M = \frac{4}{3}\pi R^3 \; x \; \rho \text{ .... (ii)}$$

Substituting (ii) in (i)

$$g = \frac{G}{R^2} \; x \frac{4}{3}\pi R^3 \; x \; \rho$$

$$= \frac{4}{3}G\pi R\rho \text{ .... (iii)}$$



When the body is a distance, d, below the earth's surface, the acceleration due to gravity, g_d is given by

$$g_d = \frac{4}{3}G\pi(R-d)\rho \text{ ....(iv)}$$

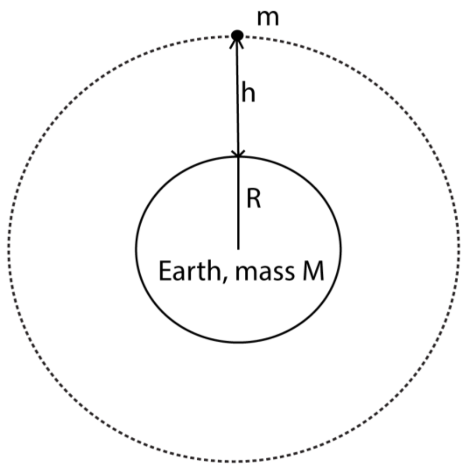
Dividing equation (iv) by (iii)

$$\frac{g_d}{g} = \frac{(R-d)}{R}\left[1 - \frac{d}{R}\right]$$

$$g_d = g\frac{(R-d)}{R}\left[1 - \frac{d}{R}\right]$$

The expression shows that acceleration due to gravity reduces as depth, d, increases toward the earth's center. At the center d = R, hence acceleration due to gravity = 0.

Variation of acceleration due to gravity with altitude



We have

$GM = R^2g$………………. (i)

$Gm = (R+h)^2g_h$ ……………(ii)

From (i) and (ii)

$R^2g = (R+h)^2g_h$

$\frac{g_h}{g} = \frac{R^2}{R^2\left[1+\frac{h}{R}\right]} = \left[1+\frac{h}{R}\right]^{-2}$

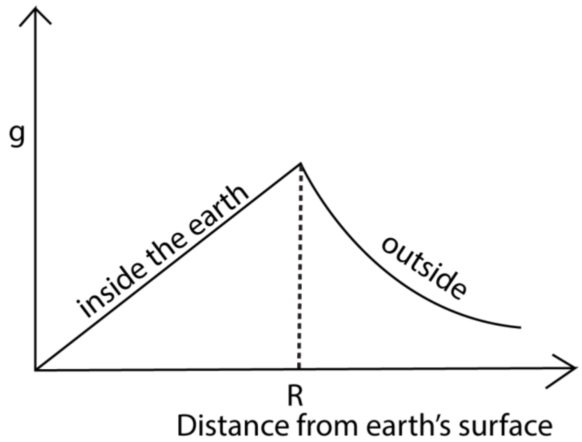
Expanding binomially and neglecting terms of higher powers of $\frac{h}{R}$ we get

$\frac{g_h}{g} = \left[1-\frac{2h}{R}\right]$

$g_h = g\left[1-\frac{2h}{R}\right]$

The expression shows that acceleration due to gravity decreases as h increases.

A graph of variation of acceleration due to gravity against the distance from earth’s center



Example

At what height will a man's weight become half his weight on the surface of the earth? Take the radius of the earth as R.

Solution

From $\frac{mg_h}{mg} = \left[\frac{R}{r}\right]^2$

$$\frac{\frac{1}{2}w}{w} = \left[\frac{R}{r}\right]^2$$

$$\sqrt{\frac{1}{2}} = \frac{R}{r}$$

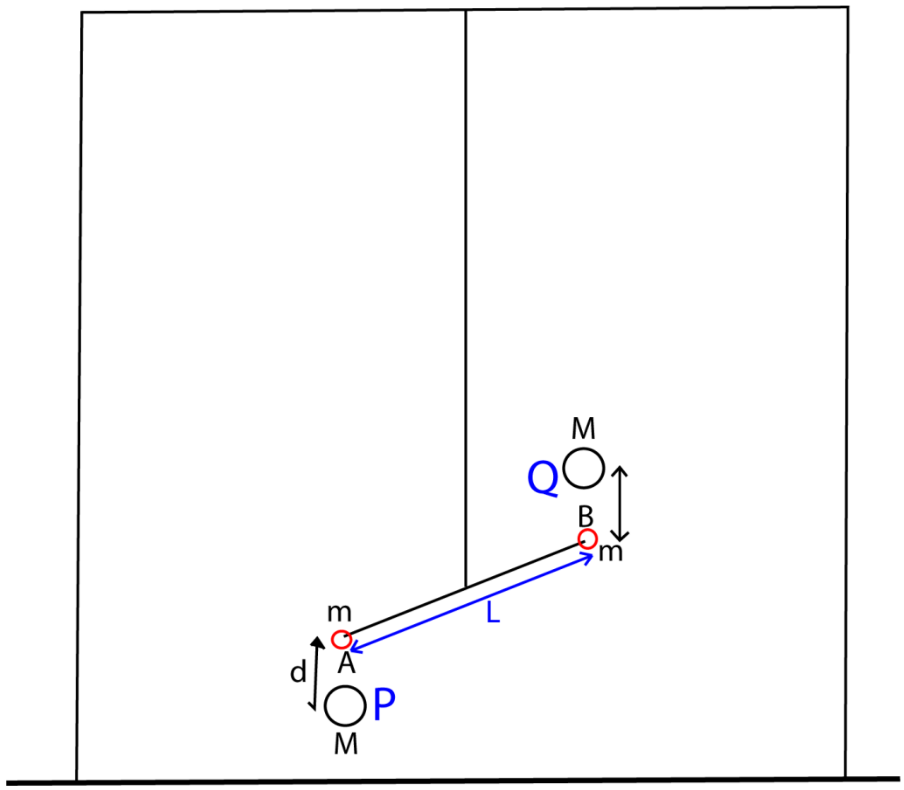
$$r = R\sqrt{2}$$

but r = (R + h)

$$R + h = R\sqrt{2} = 1.414R$$

$$H = 0.414R$$

Determining gravitational constant



- Two equal lead spheres A and B each of mass, m, are attached to end of a bar AB of length, L.
- The bar AB is suspended from a ceiling.
- Large spheres P and Q are brought towards A and B respectively from the opposite side
- Large spheres P and Q altered small spheres A and B respectively by equal and opposite gravitational forces give rise to gravitational torque, F, which in turn twist the suspended through angle θ.
- A resting torque of the wire opposes the twisting of the wire from equilibrium position

  Then

  $F = C\theta = \frac{GMm}{d^2}$

  $G = \frac{C\theta d^2}{MmL}$

  Where

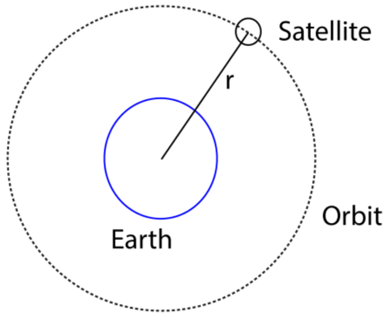
  d = distance between the center of A and P or B and Q.

  C = The twisting couple per unit twist (θ = 1)

Artificial satellite

It is a huge body that moves around a planet is an orbit due to the force of gravitation attraction. Satellite can be launched from the earth's surface to circle the earth. The are kept in their orbit the gravitational attraction of the earth.

Consider a satellite of mass, m, moving around the earth of mass M in an orbit of radius, r.



$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

But T = $\frac{2\pi r}{v}$

T = $2\pi\sqrt{\frac{r^2}{GM}}$

Parking orbital

A parking orbit is a path in space of a satellite which makes it appear to be in the same position relative to the observer at a point on the earth.

An object in a parking orbit has a period equal to the period of the earth's rotation about its axis. The direction of motion of the object in a parking orbit is in the same sense as the rotation of the earth about its axis.

The angular velocity of an object in its orbit is equal to that of the earth as it rotates about its axis. An object in a parking orbit is directly above the equator

Uses of parking orbits
Used in telecommunication

Disadvantage of parking orbits

Many geostationary satellites are required for efficient broad casting which makes very expensive.

Communication can only occur provided there is no obstruction between the transmitter and the receiver.

Gravitational Potential energy

This is the work done in moving a body of mass m from infinity to a point in the earth's gravitational field.

F = $\frac{-Gm}{r^2}$
Δw = FΔr
Total work done in moving a mass m from infinity to a point from earth of mass, M is given by

$\int \Delta w = \int_{\infty}^{r} \frac{GMm}{r^2}\,\delta r$

$\quad = \text{GMm}\int_{\infty}^{r} \frac{dr}{r^2}$

$= GMm\left[\frac{-1}{r}\right]_{\infty}^{r}$

$w = \frac{-GMm}{r}$

∴ Gravitational potential energy on the earth's surface, $w = \frac{-GMm}{r}$

Kinetic energy

Consider a body of mass, m, moving around the earth of mass, M, with velocity v.

K.E of the body = $\frac{1}{2}mv^2$

Centripetal force = $\frac{mv^2}{r}$

But gravitational force of attraction = $\frac{GMm}{r^2}$

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$

K.E = $\frac{1}{2}\frac{GMm}{r}$

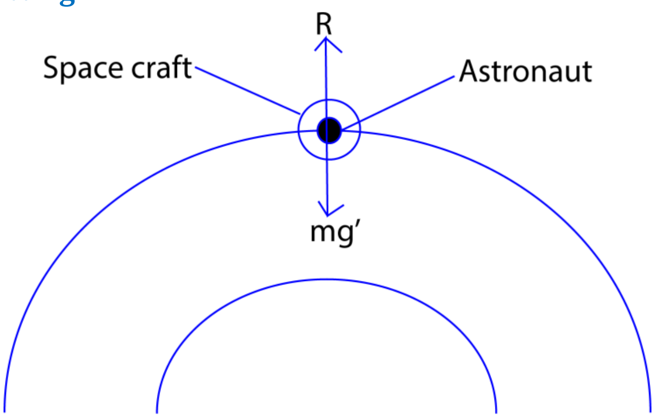
Total mechanical energy = K.E + P.E

$\frac{1}{2}\frac{GMm}{r} + \frac{-GMm}{r}$

M.E = $\frac{GMm}{2r}$

NB. For this reason, the moon is very cold. The sum has atmosphere because of the stronger gravitational attraction caused by its mass and a much higher escape velocity

Weightlessness



Let g' = acceleration due to gravity at a certain height

R = reaction of the space craft in contact with astronaut.

Consider a body moving in a space craft at a particular height of the orbit.

R = mg'

ma = mg'-R

If g' = a

mg' = ma

R = 0

Therefore the body becomes weightless

Example

A body of mass 15kg is moved from the earth's surface to a point 1.8 x 10^6m above the earth. if the radius of the earth is 6.4 x 10^6 and the mass of 6.0 x 10^{24}kg, calculate the work done in taking the body to that point.

Solution

Work done = $M\ x\ \left[\frac{Gm}{a} - \frac{Gm}{b}\right]$

a = 6.4 x 106m

b = 6.4 x 10^6 + 1.8 x 10^6 = 9.2 x 10^6m

Work done = 15 x 6.67 x 10^{-11}x 6.0 x 10^{24} x$\left[\frac{1}{6.4\ x\ 10^6} - \frac{1}{9.2\ x\ 10^6}\right]$ = 2.85 x 10^8J

Example

(a) With the aid of a diagram, describe an experiment to determine universal constant, G.
(b) If the moon moves around the earth in a circular orbit of radius = 4.0 x 108m and takes exactly 27.3days to go round once. Calculate the value of acceleration due to gravity g at the earth's surface.

Solution

(c) From $\frac{GMm}{R^2} = M\omega^2 R$

But Gm = gr_e^2 and ω = $\frac{2\pi}{T}$

$$\frac{gr_e^2}{R^2} = \frac{4\pi^2 R}{T^2}$$

$$g = \frac{4\pi^2 R^3}{T^2 r_e^2}$$

$$g = \frac{4\ x\ (3.14)^2\ x\ \left(4.0\ x\ 10^8\right)^3}{(27.3\ x\ 24x\ 3600)^2\ x\ (6.4\ x\ 10^6)^2}$$

$$g = 11.08ms^{-1}$$

Example

(a) Show how to estimate the mass of the sun if the period and orbital radius of one of its planet are known. The gravitational potential u1 at the surface of a planet of mass M and radius R is given by $U = \frac{-GM}{M}$ where G is the gravitational constant.

Derive an expression for the lowest velocity, v1, which an object of mass, M, must have of the planet if it is to escape from the planet.

(b) Communication satellites orbit, the earth in synchronous orbits. Calculate the height of communication satellite above the earth.

Solution

$$\frac{GMm}{R^2} = \frac{mv^2}{r}$$

Since v = ωr = $\frac{2\pi r}{T}$

⇨ $GM = \frac{4\pi^2 r^3}{T^2}$

$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$

But $GM = gr_e^2$

$r = \sqrt[3]{\frac{gr_e^2 T^2}{4\pi^2}} = \sqrt[3]{\frac{9.81 \, x \, (6.4 \, x \, 10^6)^2 \, x \, (24 \, x \, 3600)^2}{4\pi^2}}$

$= 4.24 \times 10^7 m$

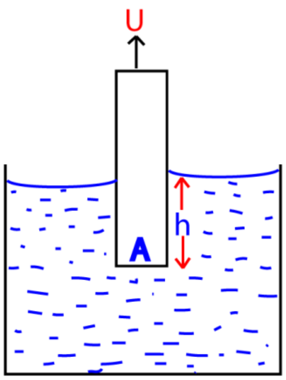
$h = r - re = 4.24 \times 10^7 - 6.4 \times 10^6 \ m$

$= 3.6 \times 10^7 m$

Pressure in liquids

This is the force exerted normally per unit area. The SI units of pressure of liquids are Nm^{-2} or Pascals (pa)

The pressure in liquids is independent of the shape and cross sectional are as shown below



Volume of a liquid displaced = Ah (A = cross section area)
Mass of the liquid displaced = Ahρ (ρ = density of a liquid)
Weight of the liquid displaced = Ahρg (g = acceleration due to gravity)
= upthrust

$$\text{Pressure} = \frac{Force}{Area} = \frac{upthrust}{area} = \frac{\text{Ah}\rho\text{g}}{A}$$
$$= \text{h}\rho\text{g}$$

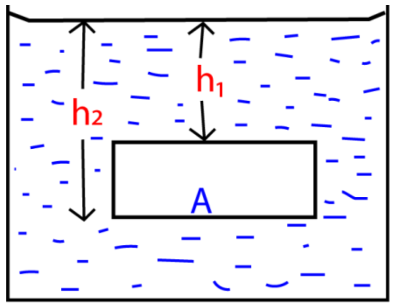
Since, ρ and g are constant; $P \propto h$

Floating objects

Archimedes principle

When a body is partially or fully immersed in a fluid, it experiences an up thrust which is equal to the weight of a fluid displaced.

Consider a solid of cross section area A immersed in a liquid of density,



Total pressure at the top = $H + h_1\rho g$
Force on top surface = $(H + h_1\rho g)A$

Total pressure at the bottom = $H + h_2\rho g$
Force on bottom surface = $(H + h_2\rho g)A$

Resultant upward force = upthrust
= $(H + h_2\rho g)A - (H + h_1\rho g)A$
= $(h_2 - h_1)\rho gA$
But $(h_2 - h_1)A$ = volume of the solid
= volume of the liquid displaced
$(h_2 - h_1)A\rho g$ = weight of liquid displaced
Upthrust = weight of the liquid displaced

Conclusion
Since there is no side way movement, the resultant horizontal force is zero. Therefore, up thrust is equal to the volume of fluid displaced.

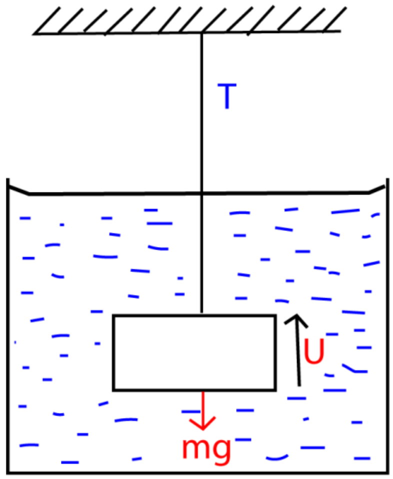
Law of floatation

States that a floating object displaces a liquid equal to its own weight oof the liquid in which it floats

Example 1

A string supports a solid of mass 5kg totally immersed in a liquid of density 800kgm^{-3}. Find the tension in the string if the object has a density of 2575kgm^{-3}.

Solution



mg = T + U

T = mg – U
U = weight of fluid displaced.
Volume of solid = volume of liquid displace

$$= \frac{Mass}{density} = \frac{5}{2575}$$

Mass of the liquid displaced = volume x density

$$= \frac{5}{2575}\ x\ 800$$

Weight of the liquid displaced, U = $\frac{5}{2575}\ x\ 800\ x\ 9.81$

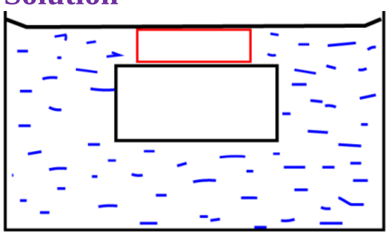
= 15.25N

Tension T = 5 x 9.81 – 15.24
= 33.81N

Example 2

A piece of metal of mass 2.60 x 10^{-3} kg and density 8.4 x 10^{3}kgm^{-3} is attached to the block of mass of 1.0x 10^{-2}kg and density 9.2 x 10^{2}kgm^{-3}. When the system is placed in a fluid, it floats with wax just submerged. Find the density of the fluid.

Solution



Volume = $\frac{mass}{density}$

Volume of liquid displaced = volume of the metal + volume of wax

$$= \frac{2.60\ x\ 10^{-3}}{8.4\ x\ 10^{3}} + \frac{1.0\ x\ 10^{-2}}{9.2\ x\ 10^{2}}$$

=1.118 x 10^{-5}m^{3}

Mass of the liquid displaced = mass of metal + mass of wax

= 2.60 x 10^{-3} + 1.0x 10^{-2}

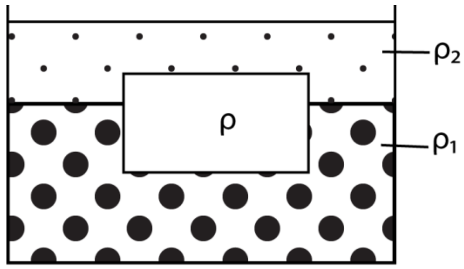
= 0.0126kg

Density of a liquid = $\frac{mass}{volume} = \frac{0.0126}{1.118\ x\ 10^{-5}}$ = 1,127kgm^{-3}

Example 3

A solid of density, ρ, floats at the interface of two liquids of densities ρ_1 and ρ_2 with 80% of its volume in liquid of density ρ_1. Show that $\frac{\rho - \rho_2}{\rho_1 - \rho} = 4\ (\rho_1 > \rho_2)$

Solution



Let the volume of the solid be V

Volume of the solid in liquid of density, $\rho_1 = \frac{8}{10}V$

Volume of the solid in liquid of density, $\rho_2 = \frac{2}{10}V$

Mass of the solid = Vρ

$$V\rho = 0.8V\rho_1 + 0.2V\rho_2$$

$$\rho = 0.8\rho_1 + 0.2\rho_2$$

$$0.8\rho + 0.2\rho = 0.8\rho_1 + 0.2\rho_2$$

$$0.2(\rho - \rho_2) = 0.8(\rho_1 - \rho)$$

$$\frac{(\rho - \rho_2)}{(\rho_1 - \rho)} = 4$$

Relative density (R.D)

This is the ratio of mass of a substance to the mass of equal volume of water.

$$R.d = \frac{mass\ of\ a\ substance}{mass\ of\ equal\ volume\ of\ water}$$

$$R.d = \frac{Weight\ of\ a\ substance}{Weight\ \ of\ equal\ volume\ of\ water}$$

$$= \frac{weight\ of\ substance}{upthrust}$$

$$= \frac{weight\ in\ air}{apparent\ losss\ in\ weight}$$

Example 4

A solid of mass 0.2kg is suspended from a spring balance when the block is immersed in water the spring reads 0.84N. When the block is immersed in a liquid of unknown density, the spring balances reads 0.95N. Find

(i) Density of the block

(ii) Density of the liquid

Solution

$$R.D = \frac{weight\ in\ air}{apparent\ losss\ in\ weight} = \frac{0.2\ x\ 9.81}{(0.2\ x\ 9.81) - 0.84} = 1.749$$

Density of the solid = 1.749 x 1000 = 1749kkm^{-3}

(i) For liquids

R.D = $\frac{loss\ in\ weight\ of\ solid\ in\ liquid}{loss\ in\ weight\ of\ solid\ in\ water} = \frac{1.962-0.95}{1.962-0.84} = \frac{1.012}{1.122} = 0.902$

Density of the liquid = 0.902 x 10000 = 902kgm^{-3}

Experiment to determine the relative density of a substance that floats in water

Apparatus: thread, spring balance, object, sinker and water

By means of a thread tied to the object, determine the weight, W_1, of an object in air using a spring balance.

Attach a sinker to the object and immerse the two in water and determine the weight, W_2, of the body and the sinker.

Determine the weight of the sinker, W_3.

R.D = $\frac{weight\ in\ air}{apparent\ losss\ in\ weight} = \frac{W_1}{W_1-(W_2-W_3)}$

Density of the substance, ρ = R.D x 1000kgm^{-3}

Experiment to determine the relative density of a liquid

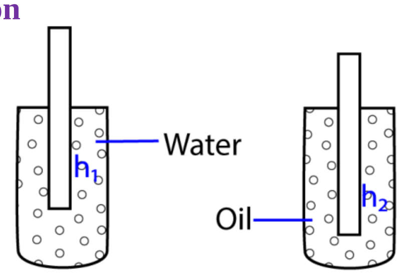
By means of a thread, determine the weight of solid in air, liquid, and water = W_1, W_2, and W_3 respectively.

R.D = $\frac{w_1 - W_2}{W_1 - W_2}$

Density rod

This is a rod used to compare densities of two liquids. The higher the rod floats, the higher the density of the liquid. A density rod floats with height h1, submerged in water. In oil, it floats with height, h2, submerged. Show that the relative density of oil is $\frac{h_1}{h_2}$.

Solution



Mass of the rod = mass of water displaced = mass of oil displaced

Let the rod have a cross sectional area = A

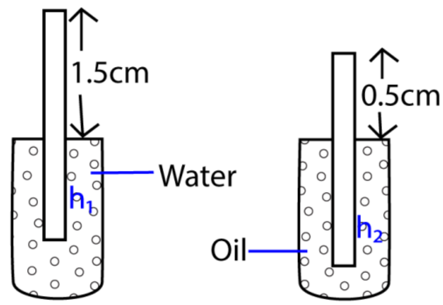
$Ah_1\rho_w = Ah_2\rho_o$

$R.D = \frac{\rho_o}{\rho_w} = \frac{h_1}{h_2}$

Example 5

An object of mass 30g and density 2g/cm^3 has a uniform cross section area of 3cm^2 floats in water and oil leaving a height of 1.5 and 0.5cm respectively above the surfaces. Calculate the relative density of oil.

Solution



$Ah_1\rho_w = Ah_2\rho_o$

$R.D = \frac{\rho_o}{\rho_w} = \frac{h_1}{h_2}$

But V = $\frac{m}{\rho} = \frac{30}{2} = 15cm^3$

V = Ah

15 = 3h

h = 5cm

h_1 = 5 – 1.5 = 3.5cm

h_2 = 5 – 0.5 = 4.5cm

R.D = $\frac{3.5}{4.5} = 0.7$

Hygrometer

It consists of a bulb that floats upright above a liquid surface. Lead shots are placed in the bulb to ensure that the system floats upright.

Example 6

A hygrometer floats on water with 72% of its volume submerged in a liquid. It floats with 68% of its volume submerged. Find the relative density of the liquid

Mass of hygrometer = mass of water displaced = mass of liquid displaced

Let the volume of hygrometer = V

$\frac{72V\rho_l}{100} = \frac{68V\rho_w}{100}$

$\frac{\rho_l}{\rho_w} = \frac{68}{72} = 0.94$

Types of flow

1. **Laminar flow**

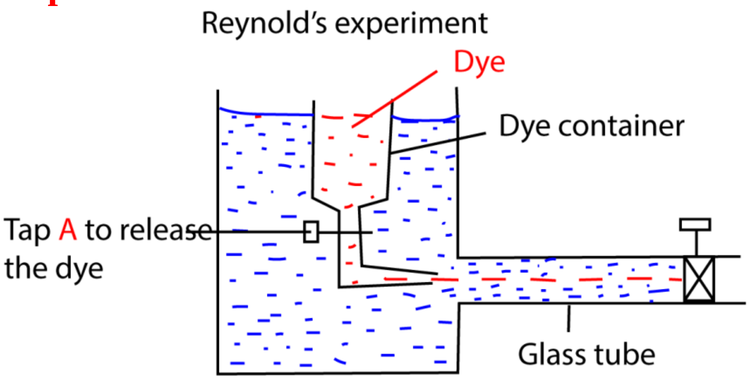
   Laminar flow occurs when the fluid flows in infinitesimal parallel layers with no disruption between them. The successive particles passing a given point have the same velocity.

   The velocity of particles may change from one streamline to another

2. **Turbulence/turbulent flow/non uniform flow**
   In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction.

   Common examples of turbulent flow are blood flow in arteries, oil transport in pipelines, lava flow, atmosphere and ocean currents, the flow through pumps and turbines, and the flow in boat wakes and around aircraft-wing tips.

Experiment to demonstrate laminar and turbulent flow



Water is kept flowing at a constant velocity from a constant water tank.

The rate of flow of a dye is controlled by a tap A.

At low water velocity a streamline of a dye is observed flowing through water. This is laminar flow

A turbulent flow is observed when the velocity of water is increased here the dye mixes with water.

Viscosity

This is the frictional force that opposes the relative motion between different fluid layers. It is the result of intermolecular forces between particles within a fluid which necessitates work to be done when layer move over one another.

Factors affecting the magnitude of viscosity

1. Temperature: increase in temperature reduces intermolecular forces due to increased kinetic energy. This reduces viscosity.

   In gases viscosity increases as temperature increases due to molecular diffusion from one layer to another of different velocities. As the temperature increases, the rate of diffusion also increases and the drag exerted on each layer by the other increases.

2. Chemical composition
   The viscosity of liquids generally depends upon the size, shape and chemical nature of their molecules.
   It is greater with larger than with smaller molecules; with elongated than with spherical molecules.
   Large amounts of dissolved solids generally increase viscosity. Small amounts of electrolytes lower the viscosity of water slightly.

3. Colloid Systems:
   The viscosity of lyophilic colloid solution is generally relatively high.

4. Suspended Material:
   Suspended particles cause an increase in the viscosity. The viscosity of blood is important in relation to the resistance offered to the heart in circulating the blood. The heart muscle functions best while working against a certain resistance. The viscosity of blood is due largely to the emulsoid colloid system present in plasma and to the great proportion of suspended corpuscles.

Velocity gradient

This is the change in velocity per unit length

Velocity gradient = $\frac{\Delta V}{L} = \frac{V}{L} = \frac{LT^{-1}}{L} = T^{-1}$

Newton's law of viscosity

The frictional force between different fluid layers is directly proportional to the area of molecular layer.

F ∝ A …………………………………… (i)

The frictional between different fluid layers is directly to velocity gradient

$F \propto \frac{\Delta V}{L}$ …………………………………… (ii)

Combining (i) and (ii)

$F \propto \frac{\Delta V}{L} A$

$F = \eta \frac{\Delta V}{L} A$

$\eta = \frac{F}{\frac{\Delta V}{L} A}$

If A = $1m^2$, $\frac{\Delta V}{L} = 1s^{-1}$, F = 1N

$\eta = 1Nsm^{-2}$

Coefficient of viscosity of a liquid

This is the frictional force per unit area exerted on the fluid in the region of a unit velocity gradient

Or

It is the ratio of tangential stress exerted on layers of fluid to velocity gradient. Units are $NM^{-2}s$ or Ns/m^2. Other units are $kgm^{-1}s^{-1}$.

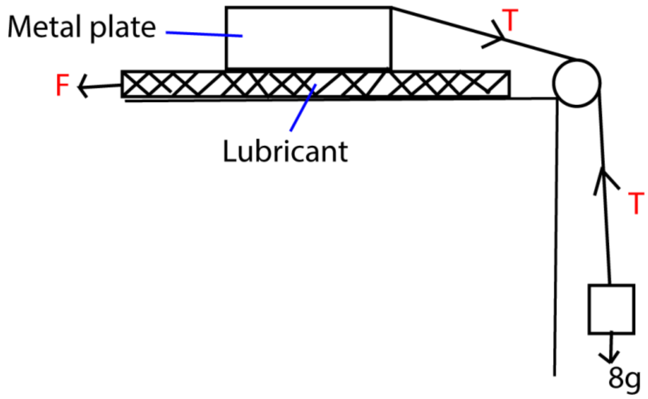
Example 7

A metal plate of area $0.25m^2$ is connected to 8g mass via a light string that passes over a frictionless pulley. A lubricant with a film of thickness 0.6mm is placed between the plate and the horizontal surface. When released, the plate moves with a speed of $87ms^{-1}$

(i) Find the coefficient of viscosity of lubricant

(ii) State any assumptions made

Solution



F = mg

$$F = \eta \frac{\Delta V}{L} A$$

$$\eta = \frac{F}{\frac{\Delta V}{L} A} = \frac{8 \times 10^{-3} \times 9.81}{\frac{(0.087-0)}{0.6 \times 10^{-3}} \times 0.25}$$

$= 2.16 \times 10^{-3} Nsm^{-2}$

(ii) the top layer of the film is assumed to move with the same velocity as the metal plate while the bottom layer is stationary.

Viscous drag

This is the frictional force that oppose relative motion between a solid and a viscous fluid

Stokes' law

The viscous drag experienced by an object depends on the velocity, viscosity constant and the radius of an object.

$F \propto v^x \eta^y r^z$

$F = k\, V^x \eta^y r^z$

$[F] = [v]^x [\eta]^y [r]^z$

$MLT^{-2} = (LT^{-1})^x (MLT^{-1}T^{-1})^y (L)^z$

For M: y = 1

For T: -2 = -x – y

x = 1

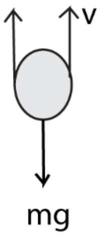
For L: 1 = x – y +z

Z = 1

From repeated experiments, k = 6π

⇨ F = 6πηvr

Example 8
The water drop of mass 10g falls through air of viscosity constant 1.0x 10^{-5}Pa. Calculate the viscous drag, experienced by the droplet when it attains a terminal velocity of 2mms^{-1}

Solution


F = 6πηvr

m = volume x density

10 x 10^{-3} = $\frac{4}{3}\pi r^3\rho$

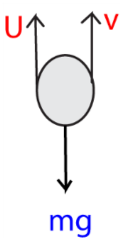
$r^3 = \frac{3 \, x \, 10 \, x \, 10^{-3}}{4\pi \, x \, 1000}$

r = 0.0134m

F = 6πηvr = 6π x (1.0 x10^{-5}) x (2 x 10^{-3}) x 0.0134

= 5.04 x10^{-4}N

Terminal velocity
Consider a spherical object dropped in a viscous fluid



As the object drops, it is acted on by three forces, U= up thrust up, viscous drag up and weight, (mg) down.

F = mg – (U+v)

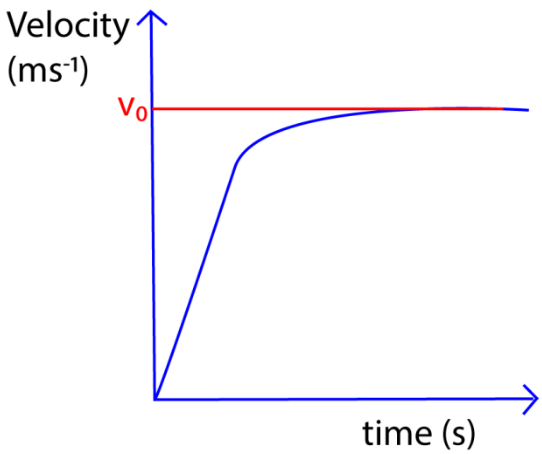
But U = 6πηv_0r

As the velocity increases, the viscous drag force increases. At a certain velocity v_0, known as terminal velocity, the resultant force acting at the body is zero.

mg = U + v

Definition
Terminal velocity is the maximum constant velocity attained by an object falling through a viscous fluid.

A graph of the velocity of an object falling through a viscous fluid against time



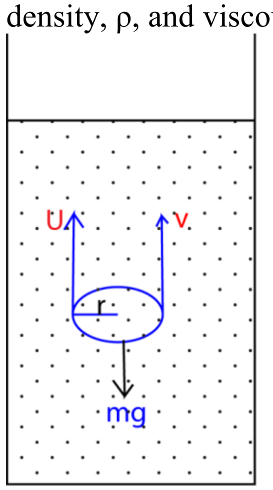
Example 9

Explain why raindrops hit the ground with less force than they should.

The drag force and up thrust reduce the force by which raindrops would hit the grounds

Derivation of terminal velocity

Consider a spherical object of radius r and density, σ, falling through a viscous fluid of density, ρ, and viscous constant, η.



mg = U + v

mg = $\frac{4}{3}\pi r^3 \sigma g$

U = weight of fluid displaced

$= \frac{4}{3}\pi r^3 \rho g$

$\frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta v_0 r$

$6\pi\eta v r = \frac{4}{3}\pi r^3 (\sigma - \rho) g$

$v_0 = \frac{2r^2(\sigma - \rho)g}{9\eta}$

Example 10

A spherical ball of radius 2.5mm and density 900kgm^{-3} fall through air of viscosity constant 1.88 x 10^{-3}.calculate the terminal velocity if

(i) Density of air is 1.29kgm^{-3}.

(ii) Density of air is negligible.

Solution

$v_0 = \frac{2r^2(\sigma-\rho)g}{9\eta}$

(i) $v_0 = \frac{2(2.5\ x\ 10^{-3})^2\ x\ 9.81\ x\ (900-1.29)}{9\ x\ 1.88\ x\ 10^{-3}}$

= 6.513ms^{-1}

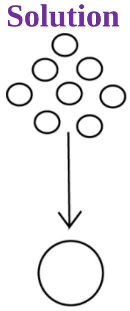
(ii) $v_0 = \frac{2(2.5\ x\ 10^{-3})^2\ x\ 9.81\ x\ 900}{9\ x\ 1.88\ x\ 10^{-3}}$

= 6.523ms^{-1}

Example 11

Eight similar water drops fall with a terminal velocity of 5mms^{-1}. And when mid-way, they coalesce forming a big droplet. Calculate the terminal velocity of a big droplet if the density of air is negligible.

Solution



$v_0 = \frac{2r^2(\sigma-\rho)g}{9\eta}$

When density of air is negligible, terminal velocity v_r for small droplets is given by

$V_r = \frac{2r^2\sigma g}{9\eta}$ = 5.0 x 10^{-3} ………………………….. (i)

Terminal velocity for big droplet v_R is given by

$V_R = \frac{2R^2\sigma g}{9\eta}$

By conserving volume

$$\frac{4}{3}\pi r^3\ x\ 8\sigma\ = \frac{4}{3}\pi R^3\ x\ \sigma$$

$R^3 = 8r^3$

R = 2r

$V_R = \frac{2(2r)^2\sigma g}{9\eta}$ ………………………………….. (ii)

Dividing Eqn (ii) with Eqn (i)

$$\frac{v_R}{5\ x\ 10^{-3}} = \frac{2(2r)^2\sigma g}{9\eta}\ x\ \frac{9\eta}{2r^2\sigma g}$$

$$v_R = 2\ x\ 10^{-2} ms^{-1}$$

Example 12

A metallic ball of mass 0.9g and diameter 8mm is dropped in oil of density 780kgm^{-3} attaining a terminal velocity of 0.07ms^{-1}. A ball falls with terminal velocity of 0.03ms^{-1} when oil is replaced with water of density 1000kgm^{-3}. Find the ratio of the coefficient of viscosity of oil to that of water at the same temperature.

Solution

Density of the ball, $\sigma = \frac{mass}{volume} = \frac{0.9 \ x \ 10^{-3}}{\frac{4}{3}\pi(4 \ x \ 10^{-3})^3}$ = 3.35x10^{3}kgm^{-3}

$$\eta = \frac{2r^2(\sigma-\rho)g}{9v}$$

For oil,

$$\eta_o = \frac{2\left(4 \ x \ 10^{-3}\right)^2 x \ 9.81(3.36 \ x \ 10^3-780)}{9 \ x \ 0.07}$$

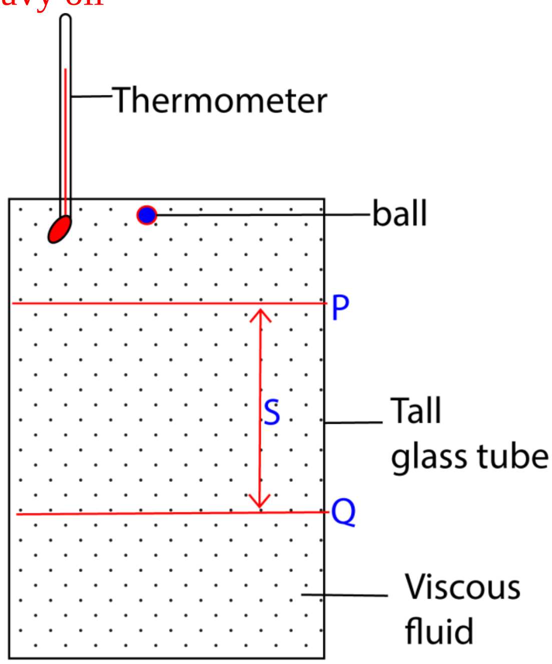
For water,

$$\eta_w = \frac{2\left(4 \ x \ 10^{-3}\right)^2 x \ 9.81(3.36 \ x \ 10^3-1000)}{9 \ x \ 0.03}$$

$$\frac{\eta_o}{\eta_w} = \frac{2\left(4 \ x \ 10^{-3}\right)^2 x \ 9.81(3.36 \ x \ 10^3-780)}{9 \ x \ 0.07} \text{ x } \frac{9 \ x \ 0.03}{2\ (4 \ x \ 10^{-3})^2 \ x \ 9.81(3.36 \ x \ 10^3-1000)}$$

$$\frac{\eta_o}{\eta_w} = \frac{2850 \ x \ 3}{7 \ x \ 2360} = \frac{12}{28}$$

Experiment to determine the coefficient of viscosity of a viscous fluid such as heavy oil

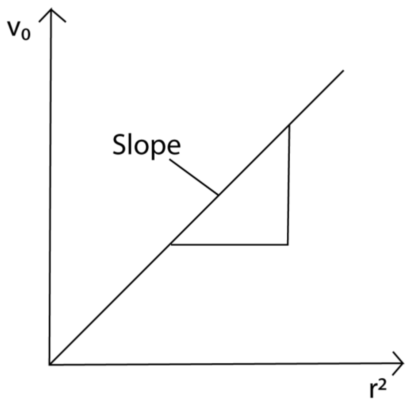


1. A viscous fluid of density, ρ, at constant temperature is put in a tall glass jar with reference marks P and Q a distance, S, apart.
2. A ball bearing of density, σ, and radius, r, is dropped into the fluid.
3. Time taken, t, taken for the ball bearing to drop from P to Q is noted

4. Assuming the ball bearing travels with a terminal velocity, v, between P and Q, then
   $v = \frac{S}{t} = \frac{2r^2(\sigma-\rho)g}{9\eta}$
   $\eta = \frac{2r^2(\sigma-\rho)gt}{9\,x\,S}$

Experiment to determine the coefficient of viscosity of a viscous liquid using the graphical method

1. A viscous fluid of density, ρ, at constant temperature is put in a tall glass jar with reference marks P and Q a distance, S, apart.
2. A ball bearing of density, σ, and radius, r, is dropped into the fluid.
3. Time taken, t, taken for the ball bearing to drop from P to Q is noted
4. Assuming the ball bearing travels with a terminal velocity, v, between P and Q, then, $v_0 = \frac{S}{t}$
5. The procedure is repeated for different ball bearing having various radii.
6. The results of t, r, v_o, r^2 are tabulated.
7. A graph of v^0 against r^2 is plotted.



Slope $=\frac{2(\sigma-\rho)g}{9\eta}$

$\eta = \frac{2(\sigma-\rho)g}{9\,x\,slope}$

Experiment to compare the coefficient of viscosity of two viscous fluids

1. A viscous fluid 1 of density, ρ_1, at constant temperature is put in a tall glass jar with reference marks P and Q a distance, S, apart.
2. A ball bearing of density, σ, and radius, r, is dropped into the fluid.
3. Time taken, t_1, taken for the ball bearing to drop from P to Q is noted
4. Assuming the ball bearing travels with a terminal velocity, v_0, between P and Q, then, $v_0 = \frac{S}{t}$.
5. Procedure 1, 2, 3, 4 are repeated for viscous fluid 2 of density ρ_2 and time t_2 to fall from P to Q is determined
   $v_2 = \frac{S}{t_2}$

   $\eta_1 = \frac{2r^2(\sigma-\rho_1)g}{9\,x\,v_1}$

$\eta_1 = \frac{2r^2(\sigma-\rho_2)g}{9\ x\ v_2}$

$\frac{\eta_1}{\eta_2} = \frac{2r^2(\sigma-\rho_1)g}{9\ x\ v_1}\ x\ \frac{9\ x\ v_2}{2r^2(\sigma-\rho_2)g}$

$\frac{\eta_1}{\eta_2} = \frac{2r^2(\sigma-\rho_1)g}{9\ x\ v_1}\ x\ \frac{9\ x\ v_2}{2r^2(\sigma-\rho_2)g}$

$\frac{\eta_1}{\eta_2} = \frac{(\sigma-\rho_1)v_2}{(\sigma-\rho_2)v_1}$

Poissulle's law

During steady flow, the rate of flow of a liquid through a pipe depends on;

(i) Coefficient of viscosity of the fluid

(ii) Radius of the pipe

(iii) The pressure gradient across the pipe

$\frac{v}{t} = k\eta^x r^y \left(\frac{P}{L}\right)^z$

$L^3T^{-3} = (ML^{-1}T^{-1})^x\ L^y\ x\ \left(\frac{MLT^{-2}}{L}\right)^z$

Solving

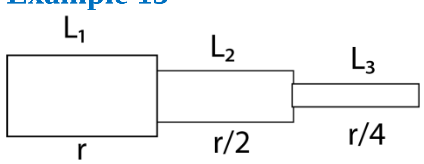
x = -1

z = 1

y= 4

$\frac{v}{t} = \frac{\pi r^4 P}{8\eta l}$, k = $\frac{\pi}{8}$

Example 13



Three pipes are arranged in some area as shown above. If the pressure in the first pipe is P1, deduce the pressure in the second and third pipe assuming there is a steady flow and $2l_1 = 3l_2 = ½\ l_3$

Solution

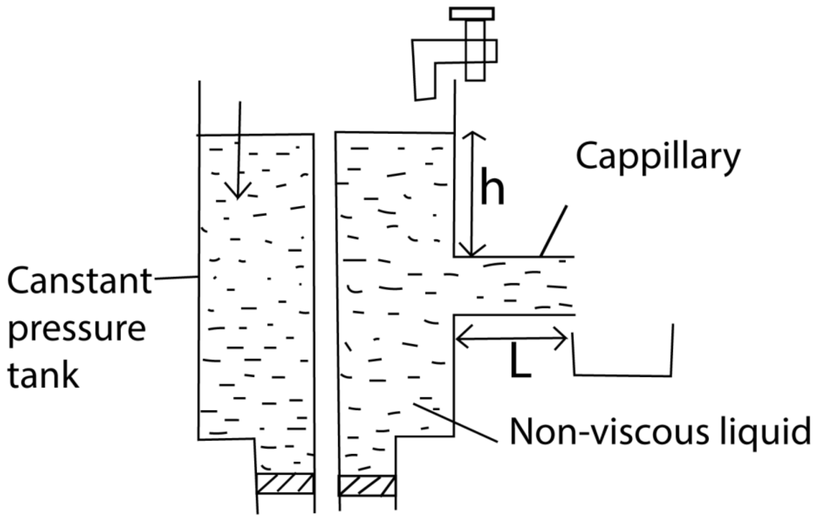
$\frac{v_1}{t} = \frac{v_2}{t} = \frac{v_3}{t}$

$\frac{\pi r^4 P_1}{8\eta l} = \frac{\pi\left(\frac{r}{2}\right)^4 P_2}{\frac{2x\ 8\eta}{3} l_1}$

$P_{2=}\frac{3P_1}{32}$

$P_3 = 1024P_1$

Experiment to determine coefficient of viscosity of non-viscous liquid



1. One end of capillary tube whose diameter, r, is known is connected to constant pressure apparatus
2. The liquid is allowed to flow in a capillary tube until a steady state is reached when height, H, is stable
3. Volume of liquid V flowing out in time t is measured.
4. The pressure height, h, in the capillary tube is measured.

For steady flow, $\frac{V}{t} = \frac{\pi r^4 P}{8\eta l}$

But P = hρg

$r = \frac{d}{2}$

$$\frac{V}{t} = \frac{\pi\left(\frac{d}{2}\right)^4 P}{8\eta l}$$

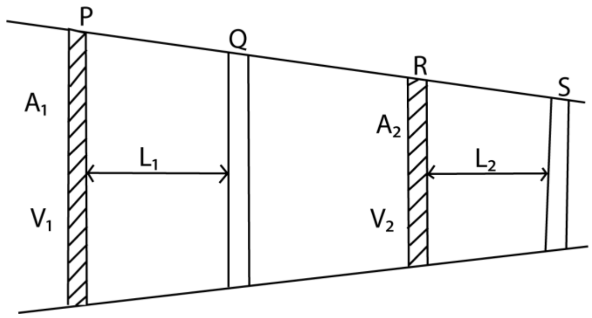
$$\eta = \frac{\pi\left(\frac{d}{2}\right)^4 \text{h}\rho\text{g}}{8\left(\frac{v}{t}\right)l}$$

Incompressible liquid

This is the liquid whose density does not change with change in pressure.

Continuity equation

Consider a non-viscous incompressible liquid flowing through a tube of non-uniform cross sectional area.



Volume of a liquid between P and Q = volume of a liquid between R and S
$A_1L_1 = A_2L_2$
But $L_1 = V_2\Delta t$
$A_1V_1\Delta t = A_2V_2\Delta t$
AV = constant
$A_1V_1 = A_2V_2$
$\frac{A_1}{A_2} = \frac{L_2}{L_1}$
$A_1 > A_2$ => L2 > L_1

$A_1V_1 = A_2V_2$
$\frac{A_1}{A_2} = \frac{V_2}{V_1}$
$A_1 > A_2$, ∴ $V_2 > V_1$

Liquids travel longer distances with high velocity in pipes of small diameter compared to those of large diameters.
$\frac{A\ x\ l}{T} = \frac{V}{T}$

$V \propto \frac{1}{A}$
For a streamline, the velocity of a liquid at any section of the pipe is inversely proportional to the area of cross section at that point

Bernoulli's principle

For a streamline flow, the sum of pressure, kinetic energy per unit volume and potential energy per unit volume is constant at all points for a non-viscous incompressible fluid.

Bernoulli's equation

A moving liquid has 3 types of energies
1. Kinetic energy: energy possessed by a liquid due to motion
2. Potential energy: energy possessed by a liquid due to its position in the field of force
3. Pressure energy: energy posed by a liquid due to its pressure at particular point.

$$P + \frac{1}{2}\rho v^2 + \rho gh = constant$$

Example 14
Explain why Bernoulli's equation only applies for a non-viscous incompressible fluid.

Solution
For the viscous fluid, energy is not constant while for compressible liquids, the density keeps on changing.

Example 15
Water enters a house through a supply pipe of diameter 2cm at a velocity of $0.1ms^{-1}$. The internal house connection pipe has the diameter of 1.0cm.
Calculate
(i) Speed of water as it enters the house
(ii) The rate of mass flow of water it it has a density of $1000kgm^{-3}$.

Solution
(i) $A_1V_1 = A_2V_2$
$\Pi(0.01)^2 \text{ x } 0.1 = \pi(0.00)^2V_2$
$V_2 = 0.4ms^{-1}$

(ii) Volume per second = AV
Mass per second = $AV\rho$
$= \pi(0.01)^2 \text{ x } 0.1 \text{ x } 1000$
$= 0.0314kgs^{-1}$

Example 16
A compound sprinkler has 8 holes each of cross sectional area of $0.05cm^2$ is connected to a supply pipe of area $2.5cm^2$. If the speed of water in the pipe is $4ms^{-1}$, calculate the speed with which water jets out of the sprinkler into the grass.

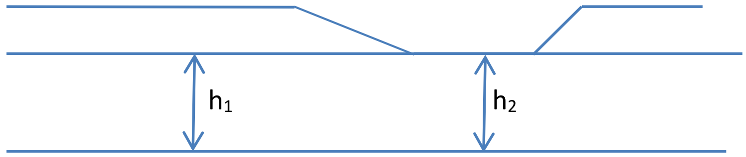
Solution

$A_1V_1 = A_2V_2$
$(2.5 \text{ x } 10^{-4}) \text{ x } 4 = 8\ (0.05 \text{ x } 10^{-4})V_2$
$V2 = 25ms^{-1}$

Example 17
Water flows along a horizontal pipe of cross sectional area $48cm^2$ with a pressure of 10^5Pa. The pipe has a constriction of area $12cm^2$ at one point. If the speed of water at the constriction is $4ms^{-1}$. Calculate

(i) speed of water in the pipe in the pipe
(ii) The pressure at the constriction

Solution
$A_1 = 48cm^2$ $A_2 = 12\ cm^2$
$V_1 = ?$ $V_2 = 4ms^{-1}$
$P_1 = 10^5Pa$ $P_2 = ?$



(i) $A_1V_1 = A_2V_2$

$(48 \times 10^{-4})V_1 = (12 \times 10^{-2}) \times 4$

$V1 = 1ms^{-1}$

(ii) From $P + \frac{1}{2}\rho v^2 + \rho gh = constant$

But $h_1 = h_2 = h$

$10^5 + \frac{1}{2} x\ 1000\ x\ 1^2 + 1000 \times 9.81h = P + \frac{1}{2} x\ 1000\ x\ 4^2 + 1000 \times 9.81h$

$P = 10^5 - 8000 + 500 = 9.25 \times 10^4 Pa$

Example 18

Water flows through a horizontal pipe with a velocity of $10ms^{-1}$ and pressure of 10^4Pa. the water flows out through a jet with pressure of 10Pa. Calculate the active velocity

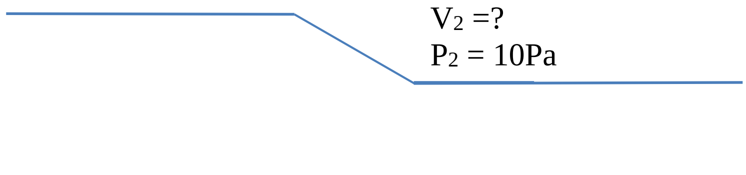
Solution

$V1 = 10ms^{-1}$

$P1 = 10^4 Pa$

$V_2 = ?$

$P_2 = 10Pa$



From $P + \frac{1}{2}\rho v^2 + \rho gh = constant$

But $h_1 = h_2 = h$

$10^4 + \frac{1}{2} x\ 1000\ x\ 10^2 + 1000 \times 9.81h = 10 + \frac{1}{2} x\ 1000\ x\ v^2 + 1000 \times 9.81h$

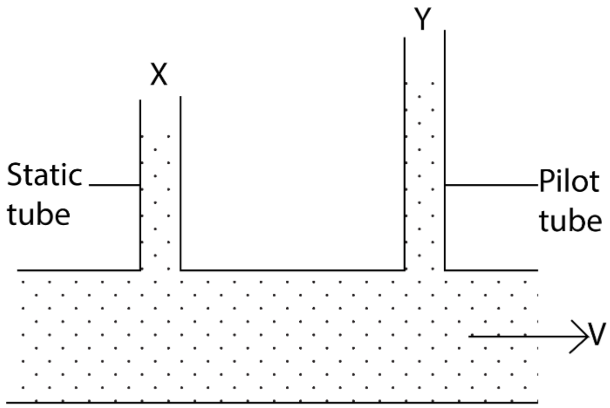
$V_2 = 10.95ms^{-1}$

Static pressure

This is the pressure exerted by a fluid at rest

Dynamic pressure

This is the pressure exerted by a fluid due to its velocity. Dynamic pressure is given by $\frac{1}{2}\rho v^2$



It consist of a static tube which measure the static pressure and the pilot tube that measures the total pressure. Total pressure is the sum of static and dynamic pressure.

From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$
Static pressure = P + ρgh
Dynamic pressure = $\frac{1}{2}\rho v^2$
Total pressure, P_y = static pressure (P_x) + dynamic pressure
= P + $\frac{1}{2}\rho v^2 + \rho gh$
For horizontal tube, h is constant
But, Total pressure, P_y = static pressure (P_x) + dynamic pressure

$$P_y = P_X + \frac{1}{2}\rho v^2$$

$$(P_y - P_x) = \frac{1}{2}\rho v^2$$

$$V = \sqrt{\left(\frac{2(P_y - P_x)}{\rho}\right)}$$

Example 19

A pilot static tube fitted with a pressure gauge is used to measure the speed of the boat at the sea. Given that the speed of the boat does not exceed $10m^{-1}$ and the density of seawater is $1050kgm^{-3}$. Calculate the maximum pressure of the gauge.

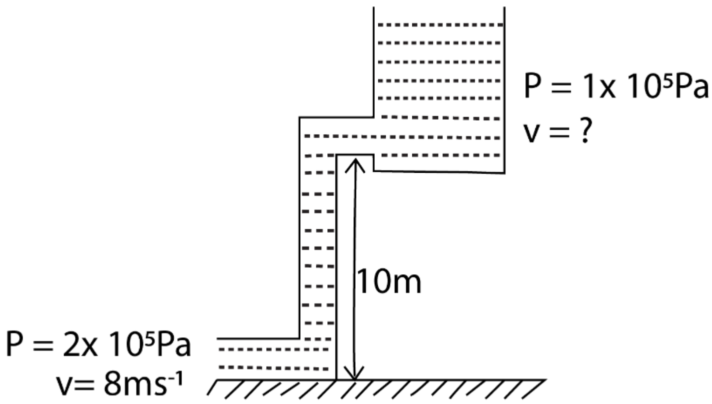
Solution

Dynamic pressure = $\frac{1}{2}\rho v^2$
= $\frac{1}{2} x\ 1050\ x\ 10^2$ = 52500Pa

Example 20

Water flowing in a pipe on aground with velocity $8ms^{-1}$ and a gauge pressure of 2×10^5Pa is pumped in a water tank 10m above the ground. Calculate the velocity with which water enters the tank at pressure of 1×10^5Pa

Solution

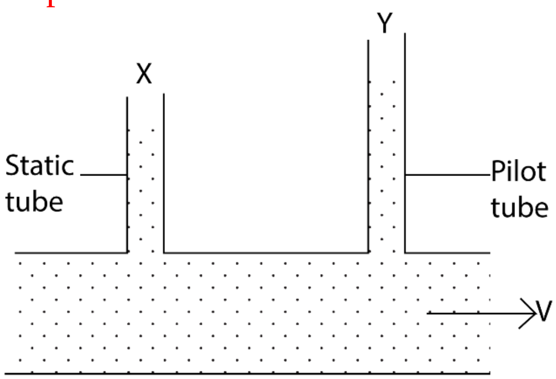


From $P + \frac{1}{2}\rho v^2 + \rho g h = constant$

$$2 \times 10^5 + \frac{1}{2} x 1000\ x\ 8^2 + 1000 \times 9.81 \times 0 = 1 \times 10^5 + \frac{1}{2} x 1000\ x\ v^2 + 1000 \times 9.81 \times 10$$

$v = 8.23ms^{-1}$

Experiment to determine the flow velocity using a pilot-static tube



A liquid of know density, ρ, is allowed to flow in the pilot static tube until the liquid levels are steady in both tubes. The total pressure, Py, is measured from the pilot tube while the static pressure is measured from the static tube.

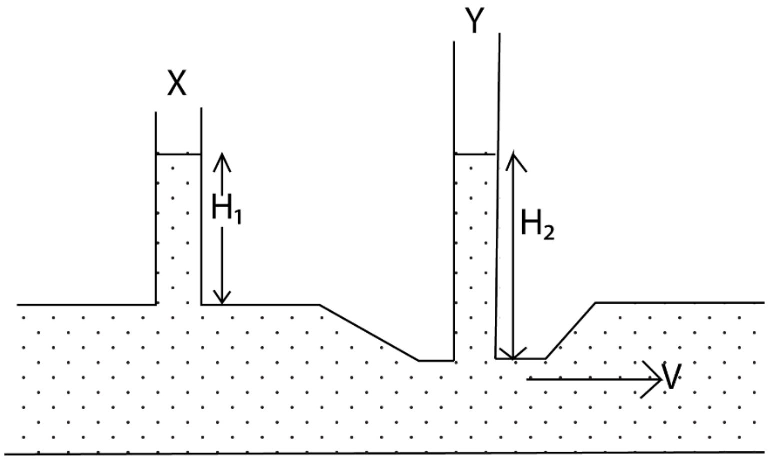
But, Total pressure, P_y = static pressure (P_x) + dynamic pressure

$$P_y = P_x + \frac{1}{2}\rho v^2$$

$$(P_y - P_x) = \frac{1}{2}\rho v^2$$

$$V = \sqrt{\left(\frac{2(P_y - P_x)}{\rho}\right)}$$

Venturimeter



This consist of the horizontal tube with a constriction at one point. Vertical manometers are inserted into the tube and at the constriction to measure respective pressures. Then

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Since $h_1 = h_2$

$P_1 = (H + H_1)\,\rho g$

$P_2 = (H + H_2)\,\rho g$

$(H + H_1) + \frac{1}{2}\rho v_1^2 = (H + H_2) + \frac{1}{2}\rho v_2^2$

$A_1V_1 = A_1V_2$

$V_1 = \frac{A_2V_2}{A_1}$

$(H + H_1) + \frac{1}{2}\rho\left(\frac{A_2V_2}{A_1}\right)^2 = (H + H_2) + \frac{1}{2}\rho v_2^2$

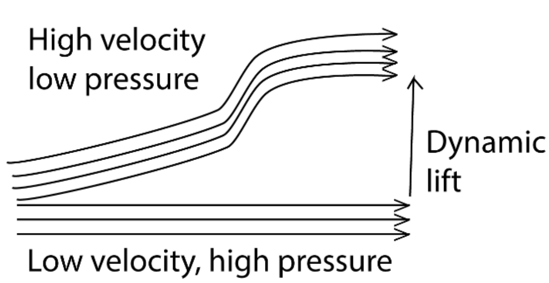
$(H_1 - H_2) = \frac{1}{2}\left(v_2^2 - \left(\frac{A_2V_2}{A_1}\right)^2\right)$

$(H_1 - H_2) = \frac{1}{2}\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$

$V = \sqrt{\frac{2(H_1 - H_2)g}{\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}}$

Application of Bernoulli's principle

1. Origin of the lift force on wings of an aero plane



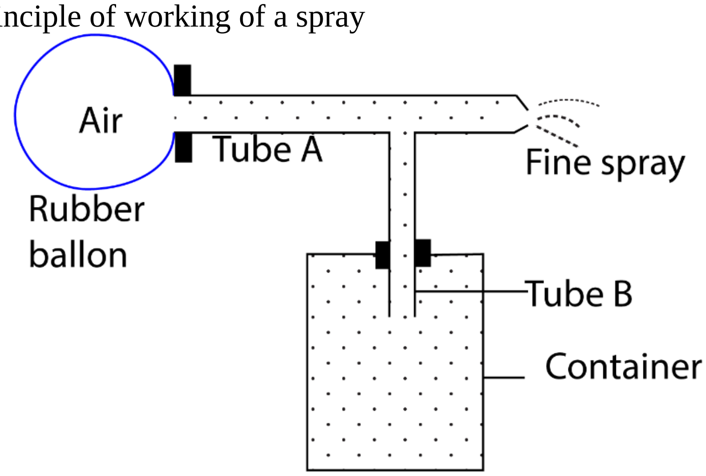
The curved nature of the wings of an aero plane ensures that at the takeoff, the air above the wing has higher velocity than that below. From Bernoulli's principle, the pressure above the wing is less than that below. This difference in pressure creates a net upward force on the wings.

Example 21
Explain why a spinning ball takes a curved path.

Solution
Air on the upper side is in opposite direction to that of the spin. The resultant velocity of the air is reduced. The air below is in the direction of the spin which increases velocity.

From Bernoulli's principle, the resultant pressure above the spinning ball is higher than that below it. The difference in pressure creates a net downward force on the ball making it to take on a curve path.

2. Principle of working of a spray



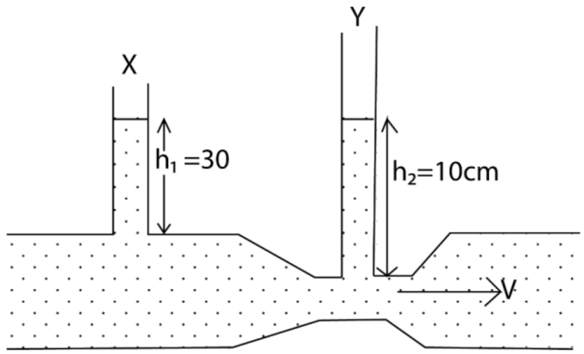
1. Pressing the rubber balloon forces the air out of the horizontal tube A at high velocity.
2. From Bernoulli's principle, this decreases the pressure in the horizontal tube to below atmospheric pressure.
3. The liquid rises up in the vertical tube B from the container.
4. The liquid collides with the fast speeding molecules of air sucked and breaks into fine spray particles.

3. Working of a vacuum cleaner

Example 22
(a) (i) Define coefficient of viscosity and determine its dimensions.
(ii) the resultant, F, on a steel ball bearing of radius, r, falling with speed, V, a liquid of viscosity, η, is given by $F = k\eta rv$, where K is a constant.

Show that K is dimensionless.

(b) Write down Bernoulli's equation for fluid flow, defining all symbols used.
(c) A venturimeter consists of a horizontal tube with a constriction which replaces part of the piping system as shown below.



If the cross sectional area of the main pipe is 5.81 x 10^{-3} m^2 and that of the constriction is 2.58 x 10^{-3} m^2. Find the velocity V_1 of the liquid in the main pipe

(d) Explain the origin of the lift on an aero plane at takeoff.

Solution

From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$

Since h is constant

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

But $P_1 =+ \rho gh_1$, P2 = $+ \rho gh_2$

$\rho gh_1 + \frac{1}{2}\rho v_1^2 = \rho gh_2 + \frac{1}{2}\rho v_2^2$

$v_2^2 - v_1^2 = 2g(h_1 - h_2)$

$A_1V_1 = A_1V_2$

$V_2 = \frac{A_1V_1}{A_2}$

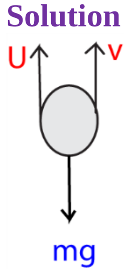
$v_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) = 2g(h_1 - h_2)$

$$V_1 = \sqrt{\frac{2(h_1-h)g}{\left(\left(\frac{A_2}{A_1}\right)^2-1\right)}} = \sqrt{\frac{2\ x\ 9.81\ (0.3-0.1)}{\left(\left(\frac{5.81\ x\ 10^{-3}}{2.58\ x\ 10^{-3}}\right)^2-1\right)}} = 0.983\text{ms}^{-1}$$

Example 23

Explain why the acceleration of a ball bearing falling through a liquid decreases continuously until it becomes zero.

Solution



For a ball bearing falling through a liquid, it has three forces acting on it namely the up thrust, U, the weight of the bearing, mg, and viscous force, v, acting as shown above.

Viscous force, v, increases with velocity reducing the accelerating force, F = mg – (U+v) which reduces the acceleration to zero when mg = (U +v).

Heat

This is a form of energy which that rises the temperature of substances.

Means of heat transfer

1. Conduction

   Conduction is the process of heat transfer through a substance from a region of high temperature to a region of low temperature without bulk movement of the medium; mainly by collision between atoms that vibrate about equilibrium positions

   Mechanism of conduction

   (a) Poor conductor

   When one end of a poor conductor is heated, the atoms at the hot end vibrate with increased amplitude, collide with neighbouring atoms and lose energy to them. The neighbouring atoms also vibrate with increased amplitude, collide with adjacent atoms and lose energy to them. In this way, heat is transferred from one end to another.

   Example 1
   Explain how heat is conducted through a glass rod (3 marks)
   Glass is a poor conductor therefore describe the mechanism of heat transfer in bad conductor.

   (b) Good conductor
   In good conductors heat is transferred from hot region to cold region by two mechanisms

   (i) Good conductors contain free electron. When heated, the electrons at hot end gain more energy, move and transfer the energy to cold region by collision with atoms in solid lattice and other electrons.

   (ii) When one end of a good conductor is heated, the atoms at the hot end vibrate with increased amplitude, collide with neighbouring atoms and lose energy to them. The neighbouring atoms also vibrate with increased amplitude, collide with adjacent atoms and lose energy to them. In this way, heat is transferred from one end to another.
   Because of the additional method of heat transfer by electrons, metals a better conductor of heat than insulators

   Example 1

Explain the mechanism of heat transfer in metals (03marks)

Mechanism of heat transfer in good conductor required

Example 3

(i) Explain the mechanism of heat transfer in solids

The question required description of conduction in both bad and good conductors

(ii) Why is a metal a better conductor of heat than glass?
Metals have additional mechanism of heat transfer by electrons.

2. Convections
Convection is a process of heat transfer in fluids from a region of high temperature to a region of low temperature, due to movement of the medium.

Mechanism of heat transfer by convection
When a fluid is heated from underneath, it expands and becomes less dense than fluid above. The warm less dense fluid rises to the top and the cool more dense fluid from above moves downwards to take place. This process continues and circulating current of the fluid is established until the whole fluid is heated up.

Example 4
(i) What is convection?
(ii) Explain how convection
3. Radiation is a means of heat transfer through a vacuum or that does not involve a medium

Temperature

Temperature is the degree hotness or coldness.

The extent to which the body feels hot depends on the; average kinetic energy of the individual atoms or molecules with in that body. This means that the body's kinetic energy is directly proportional to its thermal dynamic temperature.

Temperature is measured in degree Celsius, or Kelvin

Example 5

When is the temperature of 0K attained? (02marks)

Ok is when molecules of a substance slow down and attain their minimum total energy

Thermometry

This involves the study of thermometers as instruments used to measure temperature on the basis of certain physical thermometric properties which change with temperature and remains constant at constant temperature.

A thermometric property is a physical quantity which varies continuously, uniformly and linearly with temperature and remains constant at constant temperature.

Scales of temperature.

These are scales in which the measure of hotness or coldness of a body can be expressed .i.e. the measure of hotness or coldness of a particular body can be expressed in;

- Degrees centigrade (^{0}C) forming a Celsius scale of temperature.
- Kelvin (K) forming a thermodynamic scale of temperature.
- Degrees Fahrenheit (^{0}F) forming a Fahrenheit scale of temperature.

Convrsion os scales

If t^{0}C is temperature reading

- The thermodynamic scale is given by T= (t +273)K
- Fahrenheit scale temperature = $\left(\frac{9}{5}t + 32)\right)F$

Types of thermometers and thermometric property

Type of thermometers	Thermometric property
Liquid in glass	Length of liquid column
Electrical resistance thermometer e.g. platinum resistance	Resistance of the platinum wire
Gas thermometer e.g. constant volume gas thermometer	Changes in pressure with temperature
Thermocouple	Induced e.m.f
Radiation pyrometer	radiation

Qualities of a good thermometric property

- Considerably vary for small changes in temperature.
- Vary over a wide range of temperature (both high and low)
- Vary linearly, uniformly and continuously with temperature
- Be accurately measurable over a wide range of temperature with a simple apparatus

Related concepts

(i) **Fixed point** is defined as constant temperature at which a physical state of pure water is expected to change at 760mmHg. Fixed points are basically two i.e. 0^{0}C and 100^{0}C.

(ii) **Lower fixed point (T_0)** is the temperature of pure melting ice at 760 mmHg. It is 0^{0}C.

(iii) **Upper fixed point (T_{100})** is the temperature of pure steam at 760mmHg. It is 100^0 C.

(iv) **Triple point (T_{tr})** is the temperature at which pure water pure steam and pure ice co – exist in equilibrium at 760mmHg. It is 0.16^0 C or 273.16K.

(v) **Fundamental interval** is the range of the thermometer readings at the two fixed points e.g. for thermometers which give direct readings of temperature = 100^0 C.

Establishing scales of temperature

(i) Chose a physical property of substance (X)

(ii) Determine the value of the property at the upper fixed point of the thermometer (X_{100}). This is done by placing the bulb of the thermometer in steam at 760mmHG until a constant reading is obtained.

(iii) Determine the value of the property at the upper fixed point of the thermometer (X_0). This is done by placing the bulb of the thermometer in melting ice at 760mmHG until a constant reading is obtained.

(iv) If the value of the thermometric property at temperature, θ, is X_θ.

Then θ = $\left(\frac{X_\theta - X_0}{X_{100} - X_0}\right) x\ 100^0$C

Thermal dynamic scale of temperature

On this scale of temperature is measured in "Kelvin" or K and makes use of triple point only.

If X_{tr} is the value of a thermometric property at the triple point and X_T is the value of thermometric property at temperature, T = $\left(\frac{X_T}{X_{tr}}\ x\ 273.16\right)K$

Using a platinum resistance thermometer

T = $\left(\frac{R_T}{R_{tr}}\ x\ 273.16\right)K$

Where

- R_T is the resistance of the wire at unknown temperature, T.
- R_{tr} is the resistance of the wire at the triple point of water.

Example 6

The pressure recorded by a constant volume gas thermometer at temperature T is 4.8 x 10^4Nm^{-2}. Find T if the pressure at the triple point of water is 4.2 x 10^4Nm^{-2}.

Solution

T = $\left(\frac{P_T}{P_{tr}}\ x\ 273.16\right)K$

T = $\left(\frac{4.8\ x\ 10^4}{4.2\ x\ 10^4}\ x\ 273.16\right)K$ 312.2K

Absolute zero temperature

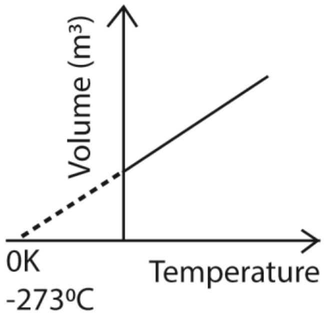
This is the temperature of an ideal gas which corresponds to its zero volume or zero pressure it exerts on the walls of the container in which it is trapped. This value approximates to the triple point of pure water i.e. - 273^0C or 0K.

Molecular explanation for existence of absolute zero temperature

When a gas is cooled, its molecules loose kinetic energy continuously since it depends directly on temperature. As molecules loose kinetic energy they move closer into close proximity until when they cease to have kinetic energy. At this point the gas is said to occupy a negligible volume and its temperature at this point is called the absolute zero temperature and the pressure the gas exerts on the walls of the container occupied is negligible.

Estimating absolute zero temperature

- Volumes a fixed mass of a gas at various temperatures are determined.
- The volumes of the gas are plotted against corresponding temperature.



Absolute zero temperature is determined by extrapolating the graph until when it touches the temperature axis and is found to be -273^{0}C or 0K.

Types of thermometers

1. **Liquid in glass thermometer**

   Depends on expansion of the liquid

   Advantages of liquid-in-glass thermometer.
   - The thermometer is simple, cheap and portable.
   - The thermometer can be calibrated to give direct readings e.g. clinical thermometer

   Liquids used in thermometers

   Mercury

   Advantages
- withstands high temperature without change of state
- does not wet glass
- expands uniformly

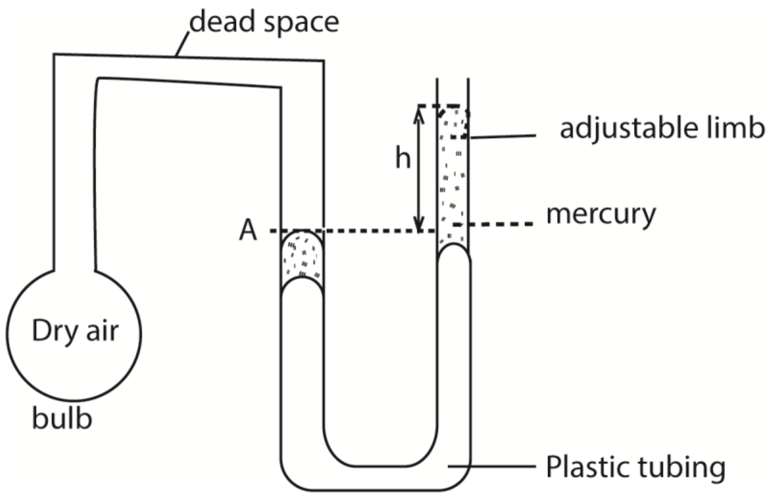
  Disadvantage
- it is expensive
- it is poisonous

  Alcohol

  Advantage
- Measures low temperatures because of its low freezing point
- Cheap

  Disadvantages
- does not measure high temperature above 78^{0}C.
- wets glass
- Water is not used because it has irregular expansion
- Cannot measure temperatures below 0^{0}Cor above 100^{0}C

2. Constant volume gas thermometer



- Place the bulb inside an enclosure whose temperature is to be measured.
- Allow some time for the gas to acquire the temperature of the enclosure. The gas in the bulb may expand and forces mercury up the adjustable tube.
- Adjust the adjustable limb to bring back mercury to constant volume at A and record the height of mercury, h_θ.
- The Celsius scale is given by θ = $\left(\frac{h_\theta - h_0}{h_{100} - h_0}\right) x\ 100^0C$ where h_{100} and h_0 are the heights at steam and ice points

Limitations

- The temperature of the gas in the dead space is different from that of the gas in the bulb
- Thermal expansion of the bulb may lead to change in volume
- Capillary effect on the mercury surface.

Correction

- The dead space should be made small
- The bulb should be made of material with low thermal expansivity
- The manometer tubes should be widened and with the same diameters.

Advantages of constant volume gas thermometers.

- It is very sensitive since a small change in temperature leads to a great expansion of a gas
- Gas thermometers give accurate results since pressure vary linearly with temperature
- Can measure wide range of temperature.

Disadvantages of constant volume gas thermometers

- it is bulky and delicate
- does not give direct readings
- it cannot measure temperature at a point
- it does not measure rapidly changing temperature

Example 7

Value of property			
Pressure in mmHg	Ice point	Steam point	Room temperature
	760	1040	795

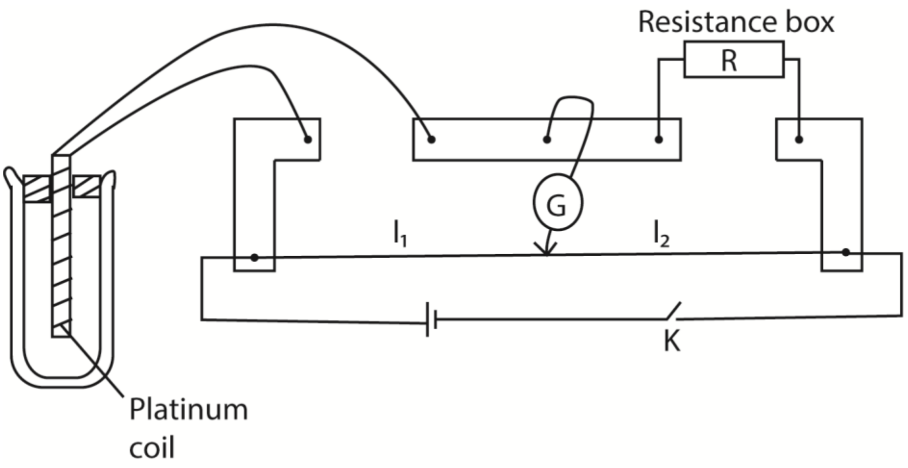
(a) Using the above data, calculate the room temperature on the scale of the constant volume gas thermometer.

θ = $\left(\frac{P_\theta - P_0}{P_{100} - Ph_0}\right) x\ 100^0C$ = $\left(\frac{795 - 760}{1040 - 760}\right) x\ 100^0C$ = 12.5^0C

(b) Explain why a gas thermometer is seldom used for temperature measurements in a laboratory.

Requires the disadvantages of the thermometer

3. Resistance thermometer



- Place the resistance thermometer in a funnel with crushed ice and leave it for some time.
- Close the switch and obtain a balance point by adjusting the resistance box,
- Determine the resistance R_0 at 0^0C from $R_0 = \left(\frac{l_1}{l_2}\right)R$
- Transfer the resistance thermometer a beaker containing boiling water and after some time, determine resistance R_{100}.
- Place the resistance thermometer in water at room temperature and determine resistance R_θ.
- Temperature of the room temperature, θ = $\left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) x\ 100^0C$

Advantages
- It is accurate
- Has fairly wide range of temperature

Disadvantages

- Cannot measure temperature at a point
- It cannot be used to follow rapidly changing temperatures

Example 8

The resistance of a certain platinum resistance thermometer is found to be 2.56Ω at 0^0C, 3.56Ω at 100^0C and 6.78Ω at 444.5^0C, the boiling point of sulphur on the gas scale

(a) Calculate boiling point of sulphur on the platinum resistance scale.

$\theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) x\ 100^0C$

$= \left(\frac{6.78 - 2.56}{3.56 - 2.56}\right) x\ 100$

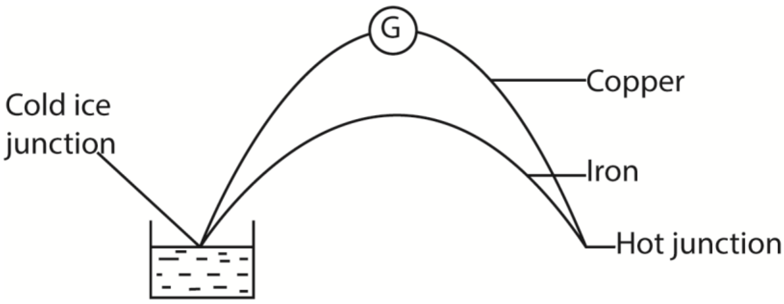
$= 422^0$C

(b) Why are the two values of boiling point of sulphur differ

Different thermometric properties in different thermometers vary differently with temperatures

4. Thermocouples

It consists of different metals such as copper and iron joined in a circuit and their junctions kept at different temperatures. This causes a small current to be produced



Mode of action

- The cold junction is put in water at 0^0C and e.m.f, E_0 is determined.
- The other junction is put at a point whose temperature is required
- As a result of difference in temperature, a thermal e.m.f and causes a deflection of the galvanometer, E_θ.
- The hot junction is placed at steam point and e.m.f, E_{100} is determined

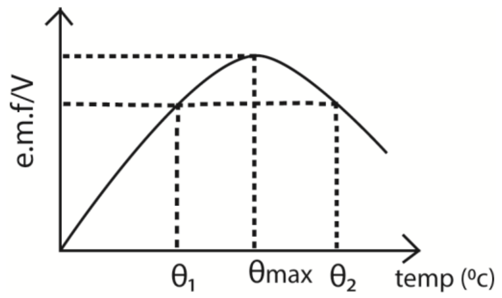
  Then $\theta^0C = \left(\frac{E_\theta - E_0}{E_{100} - E_0}\right) x\ 100^0C$

Advantages of a thermocouple

- Can be used to measure temperature at a point
- Can measure rapidly changing temperature
- Can give direct readings
- Has wide range

Disadvantages of a thermocouple

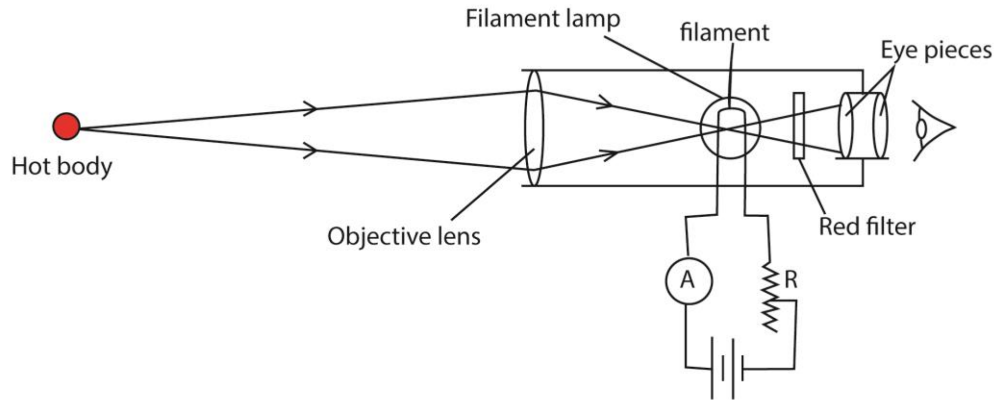
- Not accurate
- Two values of temperature θ1 and θ2 may correspond to one e.m.f



5. Pyrometers

A pyrometer is a thermometer used to measure very high temperatures by using the radiation that the body emits and wavelength of the radiation as the thermometric property.

Optical pyrometer



- the filament is focused on the eye piece and the objective focuses the object so that the image of the object lies in the same plane as the filament
- Light from the hot object and the filament is passed through the red filter and viewed by the eyepiece.
- Current is adjusted by the rheostat R until the filament and the object are equally bright.
- The temperature of the hot body is then read from the calibrated ammeter, A.

Calorimetry

Thermal equilibrium

This is a state of the body in which there is no net flow or exchange of heat within it or between it and its surroundings.

The Zeroth law of thermodynamic

It states that if two bodies are each in thermal equilibrium with a third body, then all the three bodies are in thermal equilibrium with each other.

The principle of conservation of energy

The principle of conservation of energy can be expressed mathematically by the equation

$\Delta Q = \Delta U + \Delta W$

Where

ΔQ is the quantity of heat given to the system

ΔU is the rise in internal energy, the rise in internal energy is indicated by the rise in temperature

ΔW is the external work done by the system such as expansion against atmospheric pressure.

Heat capacity

This is the quantity of heat required to raise the temperature of the body by 1^0C or 1K.

It follows that if the temperature of a body whose heat capacity C rises by $\Delta\theta$ when the amount of heat ΔQ is added to it

$\Delta Q = C\Delta\theta$ ……………………… (i)

Specific heat capacity (s.h.c), c

This is the amount of heat required to raise the temperature of 1kg mass of a substance through 1^0C or 1K.

It follows that if the temperature of a body of mass m and s.h.c, c rises by $\Delta\theta$ when an amount of heat ΔQ is added it, then

$\Delta Q = mc\Delta\theta$ …………………… (ii)

From (i) and (ii)

C = cm

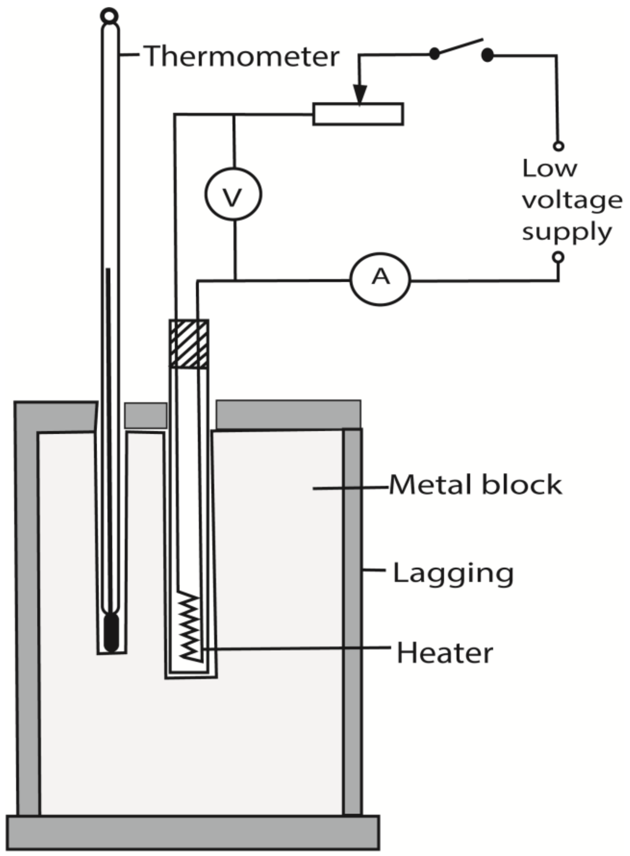
Molar heat capacity

This is the heat required to raise the temperature of 1 mole of a substance by 1Kor 1^0C.

Units: $Jmol^{-}1K^{-1}$

Measurement of specific heat capacity

(a) Measurement of specific heat capacity of solid/metal (copper, and aluminium) that are good conductors using electrical method



Simple solid block calorimeter

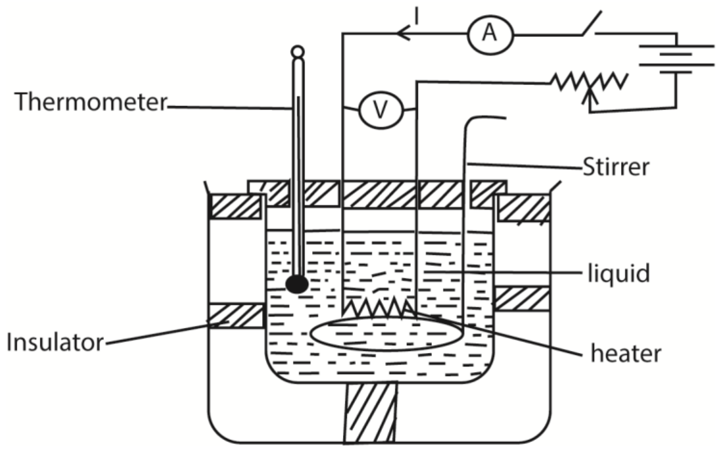
- Two holes are drilled into the specimen solid of mass m.
- A thermometer is inserted in one of the holes and an electric heater into the other hole. The holes are then filled with a good conducting fluid, e.g. oil to ensure thermal contact.
- The apparatus is insulated and initial temperature θ_0 is recorded.
- The heater is switched on at the same time a stop clock is started.
- The steady values of ammeter reading, I and voltmeter reading, V are recorded.
- After considerable temperature rise, the heater is switched off and stop clock stopped.
- The highest temperature θ_1 recorded and time t taken noted.
- Assuming negligible heat loss, the specific heat capacity, c, of the conducting solid is calculated from

  $c = \frac{IVt}{m(\theta_1 - \theta_0)}$

Precautions

- The block of metal should be highly polished and heavily lagged.
- The two holes should be filled with either mercury or light oil to improve thermal contact with the heater and thermometer.

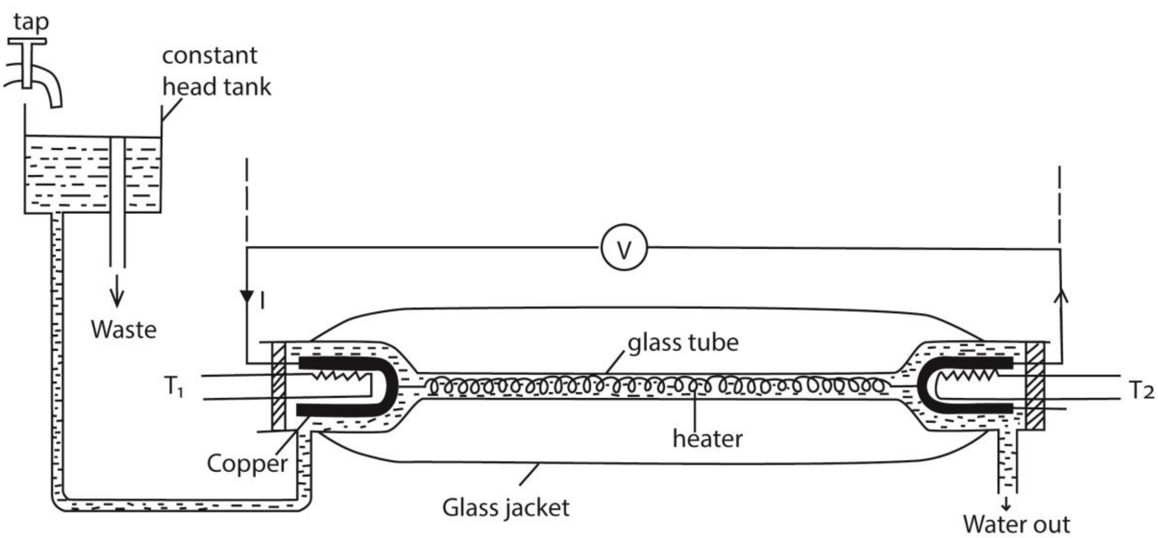
(b) Measurement of specific heat capacity of a liquid by electrical method



- A liquid of known mass, m is poured in a double walled calorimeter of a known heat capacity, C
- The setup is as shown above
- The initial temperature θ_1 of the liquid and calorimeter is recorded.
- Switch K is closed and simultaneously a top clock is started.
- The ammeter and voltmeter readings I and V are respectively recorded.
- The liquid is stirred as it is heated, and after sometime, t of heating, the current is switched off and final temperature θ_2 noted.
- Assuming no heat loss; heat supplied by the heater = heat received by water and calorimeter

  i.e. $IVt = (mc + C)(\theta_2 - \theta_1)$, where c is the specific heat capacity of a liquid.

(c) Measurement of specific heat capacity of a liquid by continuous flow method



- A liquid is allowed to flow at constant rate
- Power is switched on and the liquid is heated until temperatures registered by T_1 and T_2 are steady and the values θ_1 and θ_2 respectively are recorded.
- The p.d V and current I are recorded from the voltmeter and ammeter respectively
- The mass, m of a liquid collected in time t is recorded
- At steady state; $VIt = mc(\theta_2 - \theta_1) + h$ .... (i)

  where h is heat lost to the surrounding

- The rate of flow is changed and the voltage and current are adjusted until the steady readings of T_1 and T_2 are θ_1 and θ_2 respectively
- If m_1, V_1 and I_1 are the values mass of liquid collected in time t, voltmeter and ammeter readings respectively, then
  $V_1I_1t = m_1c(\theta_2 - \theta_1) + h$ .... (ii)
  Subtracting (ii) from (i)
  $c = \frac{(VI - V_1I_1)t}{(m - m_1)(\theta - \theta_1)}$

Example 9

In a continuous flow experiment, a steady difference of temperature of 1.5^0C is maintained when the rate of liquid flow is $4.5gs^{-1}$ and the rate of electrical heating is 60.5W. On reducing the liquid flow rate to $1.5gs^{-1}$, 36.5W is required to maintain the same temperature difference.

Calculate the

(i) Specific heat capacity of the liquid. (04marks)

$P = \frac{m}{t}c\theta + h$

$60.5 = 4.5 \times 10^{-3} \times c \times 1.5 + h$ .... (i)

$36.5 = 1.5 \times 10^{-3} \times c \times 1.5 + h$ .... (ii)

Subtracting (ii) from (i)

$24 = 3 \times \times 10^{-3} \times c \times 1.5$

c = 5,333Jkg

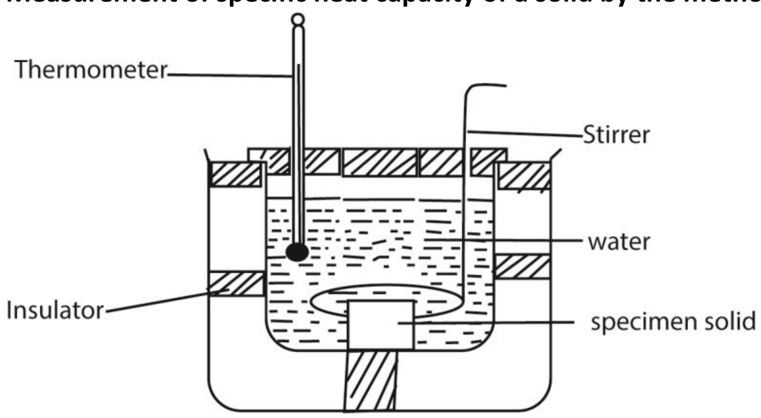
(ii) Rate of heat loss to the surroundings (03marks)

Substituting c in (i)

$60.5 = 4.5 \times 10^{-3} \times 5333 \times 1.5 + h$

h = 24.5W

(d) Measurement of specific heat capacity of a solid by the method of mixtures



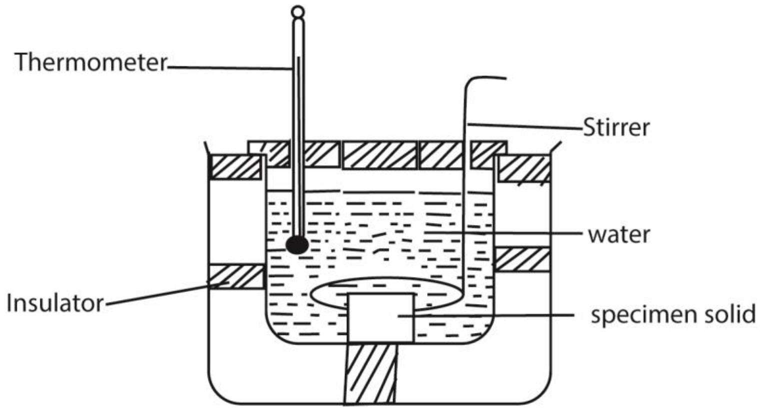
- A solid on mass m_s kg and specific heat capacity, c_s, is heated in boiling water at temperature at temperature $\theta_1{}^0C$ and quickly transferred to a calorimeter of heat capacity, C, containing water of mass, m_1 and , at the temperature θ_2.
- The final constant temperature θ_3 of the mixture is determined.
  Assuming there is no heat loss
  Heat lost by the solid = heat gained by calorimeter + heat gained by water
  $m_s \times c_s \times (\theta_1 - \theta_3) = (C + c_w m_1)(\theta_3 - \theta_2)$

  $c_s = \frac{(C + c_w m_1)(\theta_3 - \theta_2)}{m_s c_s}$ where c_w is specific heat capacity of water

Precautions

- The calorimeter must be heavily lagged.

(e) Measurement of specific heat capacity of a liqid by the method of mixtures



- A solid on mass m_s kg and specific heat capacity, c_s, is heated in boiling water at temperature at temperature $\theta_1{}^0$C and quickly transferred to a calorimeter of heat capacity, C, containing a liquid of mass, m_1 and , at the temperature θ_2.
- The final constant temperature θ_3 of the mixture is determined.
  Assuming there is no heat loss
  Heat lost by the solid = heat gained by calorimeter + heat gained by water

$m_s \times c_s \times (\theta_1 - \theta_3) = C(\theta_3 - \theta_2) + c_l m_l(\theta_3 - \theta_2)$

$c_l = \frac{(m_s\, x\, c_s\, x\, (\theta_1 - \theta_3)) - (C(\theta_3 - \theta_2)}{m_l(\theta_3 - \theta_2)}$ C_l = specific heat capacity of the liquid.

Precautions

- The calorimeter must be heavily lagged.

Example 10

Apiece of copper of mass 100g is heated to 100^0C and then transferred to a well-lagged copper can of mass 50g containing 200g of water at 10^0C. Neglecting heat loss, calculate the final steady temperature of the of the mixture.

[Specific heat capacity of copper and water are 400Jkg^{-1}K^{-1} and 4200Jkg^{-1}K^{-1})

Solution

Let the final temperature be θ.

Heat lost by the copper mass = heat gained by the copper can + heat gained by water

$0.1 \times 400 \times (100 - \theta) = 0.05 \times 400 \times (\theta - 10) + 0.2 \times 4200 \times (\theta - 10)$

$\theta = 14.0^0$C

Example 11

(i) State two advantages of the continuous flow method over the method of mixtures. (01mark)

- No cooling correction is required

- Heat capacity of the apparatus is not requires
- Temperature measured at leisure when steady
- Resistance of the heater not required

(ii) State two disadvantages of the method in (c)(i) (01mark)

- larger volumes of liquid required
- not suitable for volatile liquids.

Newton's law of cooling

Newton's law of cooling states that under forced convection, the rate of loss of heat of a body is directly proportional to its excess temperature over that of the surrounding.

i.e.$\frac{dQ}{dt} \propto (\theta - \theta_R)$

where $\frac{dQ}{dt}$ is the rate of heat loss, θ = body's temperature, θ_R = temperature of the surroundings.

The rate of heat loss also depends on

- Surface area of the body and,
- Nature of the surface, i.e. whether dull or shiny

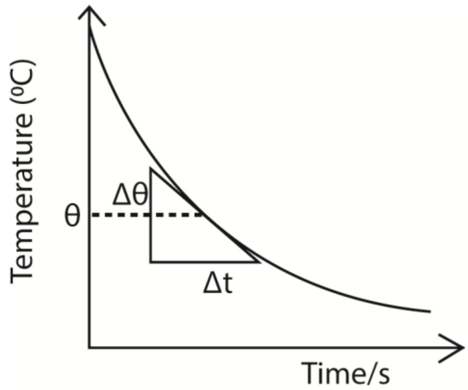
Hence for a body of a uniform temperature, θ and surface area A

Rate of heat loss, $\frac{dQ}{dt} = kA(\theta - \theta_R)$ where k is a constant.

NB. A small body cools faster than a big body because a small body has a larger surface area to volume ration.

An experiment to verify Newton's law of cooling. (05marks)

- Hot water is placed in a calorimeter that is standing on an insulating surface and is put in a draught.
- The temperature, θ, of the water is recorded at suitable intervals.
- The room temperature θ_R is recorded.
- Plot a graph of temperature against time to get a graph similar to the one below.



- Draw tangent at various temperatures, θ and obtain their slopes. These slopes give the rate of temperature fall.

- Plot these slopes with corresponding excess temperatures ($\theta - \theta_R$)
- A straight line graph is obtained implying that the rate of heat loss is proportional to excess temperature.

Example 12

Describe an experiment to verify Newton's law of cooling. (05marks)

Cooling correction

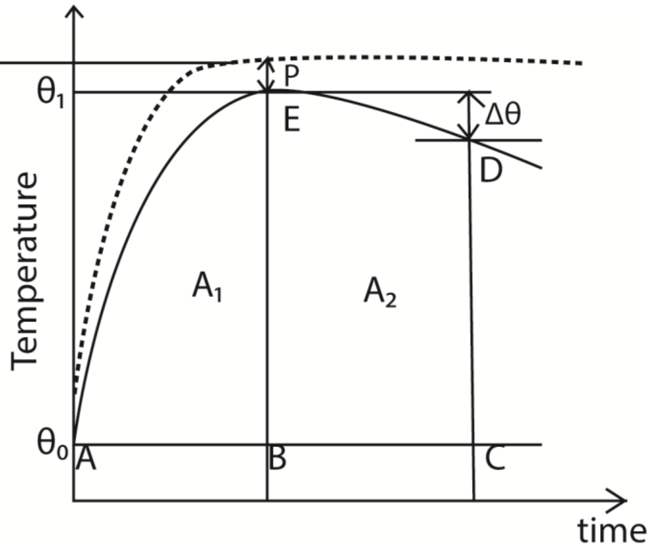
Despite all precautions to prevent heat losses in experiments of the mixtures, still there will be some significant heat loss that will prevent the mixture to attain maximum temperature rise. An estimate has to be made of the temperature that would have been attained if these heat losses were not there. The small temperature added to the observed maximum temperature to make up for the heat lost to the surrounding during the experiment is called cooling correction.

Definition of cooling correction

This is the extra temperature difference to be added to the observed maximum temperature of the mixture to make up for the heat lost to the surrounding during the experiment.

Experiment to determined cooling correction

- Pour a liquid in a calorimeter and place it on a table
- Place a thermometer into the liquid and after sometime, record the temperature of the surroundings, θ_0.
- Gently place a hot solid into the liquid and stir.
- Record the temperature of the mixture at suitable interval until the temperature of the mixture has fallen by about 1^0C below the observed maximum temperature, θ_1.
- Plot a graph of temperature against time



- The broken line shows how we would expect the temperature to rise if no heat were lost and the difference, P, between the plateau of this imaginary curve, and the crest of the experimental curve, E. is known as the 'cooling correction'
- Draw a line AC through θ_0 parallel to the time axis.
- Draw a line BE through θ_1 parallel to the temperature axis.
- Draw a line CD beyond BE parallel to the temperature axis and note $\Delta\theta$
- Estimate the area A_1 and A_2 under the graph by counting the square on the graph paper
- Cooling correction, P s given by the graph

  Cooling correction, P = $\frac{A_1}{A_2}\ x\ \Delta\theta^0$C

Example 13

i) What is meant by cooling correction? (01marks)

This is the extra temperature difference to be added to the observed maximum temperature of the mixture to make up for the heat lost to the surrounding during the experiment.

(ii) Explain how the cooling correction may be estimated in the determination of the heat capacity of poor by the method of mixtures (05marks)

Latent heat

Definition

This is the quantity of heat absorbed or released when a substance changes physical state at constant temperature. E.g. during melting, evaporation, sublimation, condensing, solidification

Example 14

Explain why there is no change in temperature when a substance is melting (04marks)

Supply of heat to a melting solid reduces the forces of attraction between the molecules and increases the separation between them. This increases the potential energy (P.E) between the molecules while keeping kinetic energy (K.E) of the molecules the same. Further increase in separation between the molecules causes the regular pattern to collapse as the solid changes to liquid. Until this process is complete, the temperature does not change.

Example 15

Explain the changes that take place in the molecular structure of substances during fusion and evaporation (04marks)

Heat supplied during fusion breaks down the forces that keep ordered pattern of molecules is solid crystalline structure to form a liquid. The potential energy of the molecules increase but the average kinetic energy and temperature of the molecules remain unchanged.

Heat supplied during evaporation breaks molecular bonds in liquids and allow gas molecules to expand against atmospheric pressure which allows then to move independently.

Example 16

Explain why the specific latent heat of evaporation is always greater than specific latent heat of fusion of a substance at the same pressure are different. (04marks)

Change from solid to liquid, intermolecular bonds are weakened and there is a small increase in volume. This implies there negligible change in volume and thus little work done against atmospheric pressure.

During vaporization, a lot of heat is required to break molecular bonds in a liquid and to enable expansion to larger volume of a gas against atmospheric pressure.

Specific latent heat of fusion

Specific latent het of fusion is the amount of heat required to change 1kg mass of a substance from solid to liquid without change of temperature. Units are Jkg-1

Experiment to determine the specific latent heat of fusion by the method of mixtures

- A known mass m_1 kg of water at temperature θ_1 (a few degrees above room temperature) is placed in a calorimeter of mass m_c kg and heat capacity. C.
- A small piece of dry ice (dried by blotting paper) is added to the water and stirred to a constant temperature θ_2.
- The total mass, m, of the calorimeter and water and ice is determined.

Calculation

Mass of ice = (m – (m_1 + m_c) = $m_3$kg

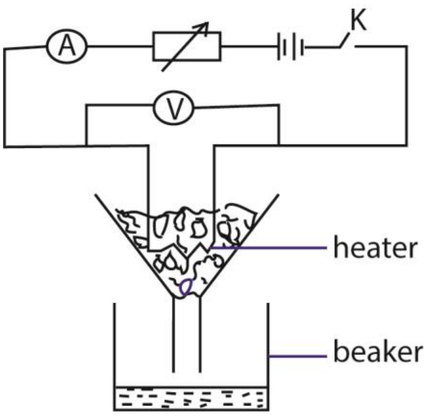
Heat lost by water and calorimeter = heat gained by ice to melt and again temperature to θ_2.

Suppose c and l_f are the specific heat capacity and latent heat of fusion of water, then

(m_1c +)($\theta_1 - \theta_2$) = m_3l_f + $m_3c(\theta_2 - 0)$

$$l_f = \frac{\left(((m_1c+C)(\theta_1 - \theta_2)) - (m_3c(\theta_2 - 0))\right)}{m_3}$$

Experiment to determine the specific latent heat of fusion by electrical method



- Place pure ice in a funnel such that it submerge the heater
- With R set, close K and simultaneously start a stop clock.
- Collect a known mass m of water in a specified time, t.
- Record the readings I and V of the ammeter and voltmeter spectively.

  Assuming no heat gain from the surrounding

  Heat supplied by the heater = heat gained by ice.

  IVt -= ml_fb where l_f is the specific latent heat of fusion

NB: if heat lost gained from the surrounding is h

Then IVt + h = mlf …………………. (i)

The experiment is repeated for different values of V', I' and m' in the same time

I'V't + h = m'lf …………………. (ii)

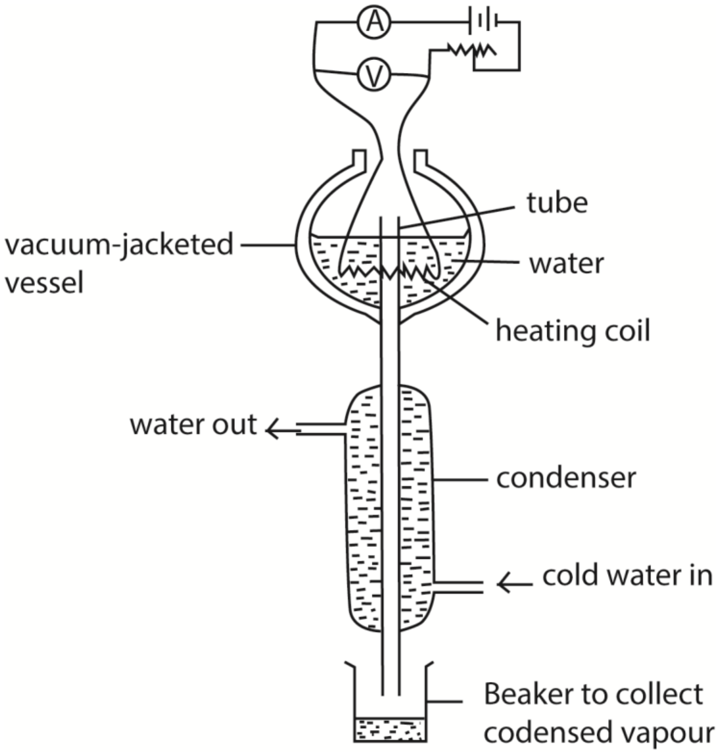
Combining (i) and (ii)

$$l_f = \frac{(IV - I\prime V\prime)t}{(m - m')}$$

Specific latent heat of vaporization

Specific latent het of vaporization is the amount of heat required to change 1kg mass of a substance from liquid to vapour without change of temperature. Units are Jkg-1

Experiment to determine the specific latent heat of vaporization of water by electrical method



- Put the liquid whose specific latent heat of vaporization is required in a vacuum jacketed vessel as shown above.
- The liquid is heated to boiling point.
- The current, I, and voltage, V are recorded.
- The mass of condensed water, m, condensed in time, t, is determined.
- Then IV = $\frac{m}{t}L + h$,

  where h is the rate of heat loss to the surroundings
- To eliminate, h, the experiment is repeated for different values of I' and V' and the mass of the condensed water, m' condensed in tie t is determined.
- Again I'V' = $\frac{m'}{t}L + h$

  Latent heat of vaporization, L =$\frac{(I'V'-IV)t}{(m'-m)}$

Example 17

An appliance rated 240V, 200W evaporates 20g of water in 5 minutes. Find the heat loss if specific latent heat of vaporization is 2.26 x 10^6Jkg^{-1}. (03marks)

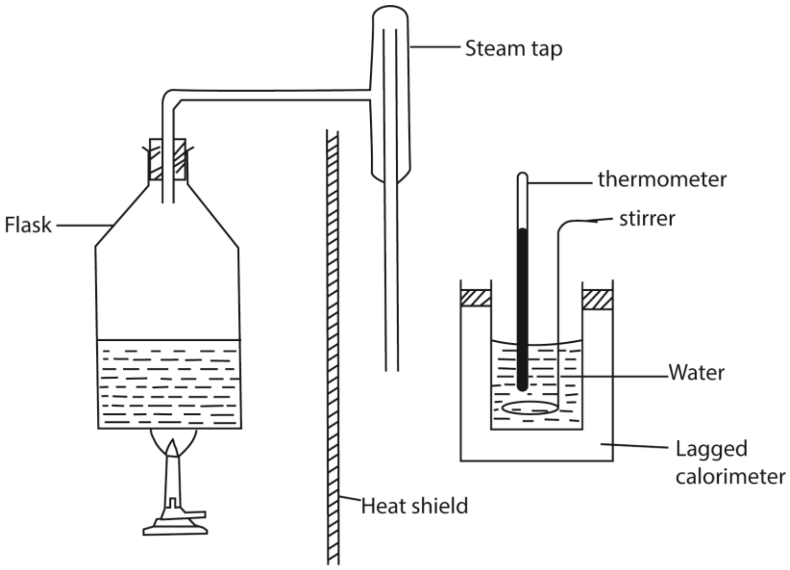
Electrical energy supplied = mlv + h

Power x time = mlv + h

200 x 5 x 60 = 20 x 10^{-3} x 2.26 x 10^6 + h

h = 14800J

An experiment to determine the specific latent heat of vaporization of water by the method of the mixtures



- The initial temperature θ_0 and mass, m of water in the calorimeter are measured
- Steam from boiling water is passed into water in a calorimeter and after a reasonable temperature rise, flow of steam is stopped and final temperature, θ_f is recorded.
- Mass m_2 of water in the calorimeter is then taken
- The mass of steam condensed, $m_s = (m_2 - m)$

  Given that the heat capacity of the calorimeter= C

  Heat gained by steam = heat gained by water and calorimeter

  $m_s c_v + m_s c(100 - \theta_f) = (m_2 - m)c(\theta_f - \theta_0) + C(\theta_f - \theta_0)$

  c_v = specific latent heat of vaporization

  c= specific heat capacity of water

Example 18

Explain why evaporation causes cooling? (03marks)

When a liquid evaporate molecules with high kinetic energy escape leaving molecules with low kinetic energy. Since temperature of the liquid depends on the average kinetic energy of its molecules, the temperature drops.

Example 19

Explain the effect of pressure on the boiling point of a liquid. (02marks)

- Since a liquid boils when its saturated vapour pressure is equal to external pressure.
- Increasing the external pressure increases the boiling point of a liquid because the liquid has to be heated to a higher temperature to make its saturated vapour pressure equal to external pressure

Gas laws and heat capacity

Gases

Definition

A gas is a term applied to a substance which is in the gaseous phase above its critical temperature.

A critical temperature is a temperature above which a gas cannot be liquefied no matter how great the pressure is.

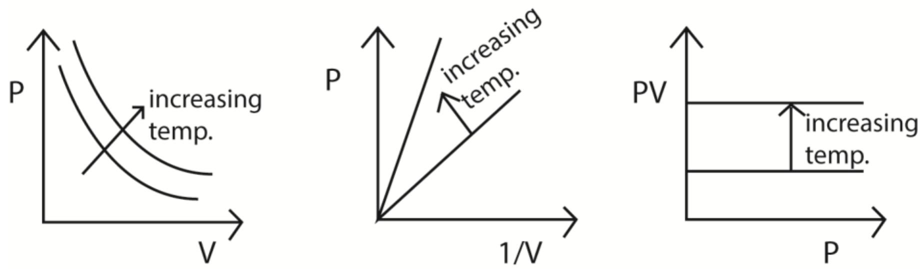
Gas laws

(a) Boyle's law

States that the pressure of a fixed mass of a gas is inversely proportional to volume. If P stand for pressure and V for volume; then

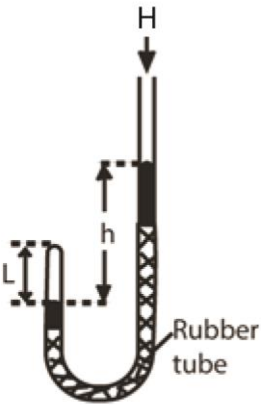
$P \propto \frac{1}{V}$ or $P = k\frac{1}{V}$ or $PV = k$ where k is a constant that depend on the mass of a gas and temperature.

Graphically



In general. When the pressure and volume of a gas change from P_1 and $V_1$to P_2 and V_2 respectively at constant temperature, then $P_1V_1 = P_2V_2$.

An experiment that can be used to verify Boyles' law.



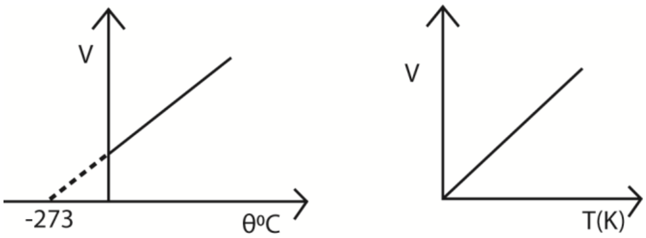
- air in the closed limb of a U-tube barometer as shown above
- Mercury is poured to a height, h, and the length of the air column, L is noted.
- the length h is varied to obtain different sets of values of h and L
- Pressure of the gas is calculated from P = (H + h)ρg where H= height of barometer corresponding to atmospheric pressure, ρ = density of mercury, g = acceleration due to gravity. Note that h can be positive or negative.
- If A is the cross section area, V =AL
- Values of h, L, P, V and 1/V are tabulated

- A plot of P against 1/V gives a straight line through the origin which verifies Boyle's law.

(b) Charles' law

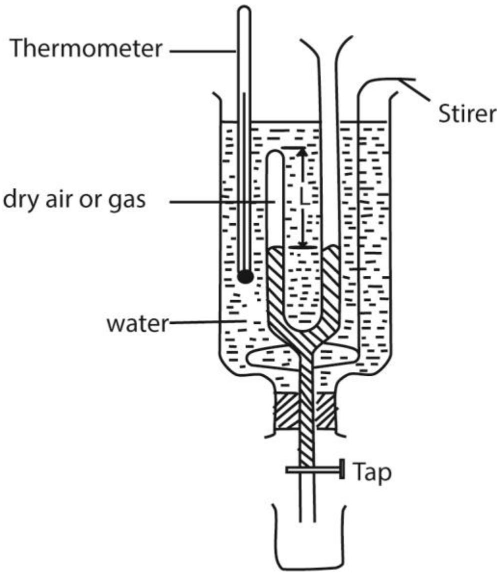
Charles's states that the volume of a fixed mass of a gas is directly proportional to its absolute temperature at constant pressure

A graph of volume against temperature is shown below



In general when the volume and temperature of a gas change from V_1 and T_1 to V_2 and T_2 respectively at constant pressure, then, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Experiment to verity Charles' law



- Dry air is trapped in the closed limb as shown above
- The level of mercury in the two limbs is maintained at the same level by adding or removing mercury at each temperature of the bath to ensure that pressure of the air is equal to atmospheric temperature.
- The length L of the air column and temperature of the water bath θ are recorded.
- Several values of L and θ are obtained by passing steam
- A graph of L against θ gives a straight line showing that the volume of the gas is proportional to temperature.

(c) **Pressure law**

It states that the pressure of a fixed mass of a gas is proportional to temperature provided the volume is constant. i.e. , $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Ideal gas equation or equation of state

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Or

PV = kT

Example 20

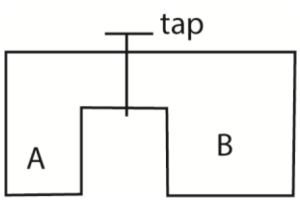
A gas cylinder has volume of $0.040m^3$ and contains air at pressure of 2.0MPa. Assuming that the temperature remain constant, calculate

(a) The equivalent volume of air at atmospheric pressure of 1.0×10^5Pa
From PV = constant
$0.04 \times 2 \times 10^6 = V \times 1.0 \times 10^5$
$V= 0.8m^3$

(b) The volume of air, at atmospheric pressure, which escapes from the cylinder when it is opened to atmosphere.
$0.8 - 0.04 = 0.76m^3$
i.e. air escapes from the cylinder until it contains $0.04m^3$ of air at atmospheric pressure

(d) Dalton's law of partial pressures
States that the total pressure of a mixture of gases, which do not react chemically is equal to the sum of the partial pressures of the gas.

Example 21



Two cylinders A and B of volumes V and 3V respectively are separately filled with a gas. The cylinders are connected as shown above with the tap closed. The pressures of A and B are P and 4P respectively. When the tap is opened the common pressure becomes 60Pa. Assuming isothermal conditions find the value of P. (04marks)

Solution

From PV = nRT

Moles n_1 of the gas in A before mixing= $\frac{PV}{RT}$

Moles n_2 of the gas in B before mixing = $\frac{4P\ x\ 3V}{RT} = \frac{12PV}{RT}$

Moles n_3 of the gas when tap is opened = $\frac{60\ x\ 4V}{RT}$

But moles of the gas before mixing = mole of the gas after mixing

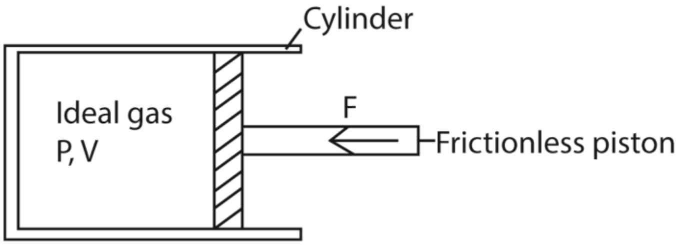
⇨ $n_1 + n_2 = n_3$

$$\frac{PV}{RT} + \frac{12PV}{RT} = \frac{60\ x\ 4V}{RT}$$

$$13P = 240$$

P = 18.46Pa

External work done by expanding gas.



Suppose the gas expands by dv so that the piston moves out through a small distance dx.

Work done by the gas, dW = Fdx

= PAdx

= Pdv

Total work done during expansion from v_1 to v_2 is given by

$$W = \int_{v_1}^{v_2} Pdv$$

Example 22

When 1.5kg of water is converted to steam at (100^0C) at standard pressure ($1.01 \times 10^5 Nm^{-2}$) 3.39MJ of heat is required. During the transformation from liquid to vapour, the increase in volume of water is $2.5m^3$.

(i) Calculate the work done against the external pressure during the process of evaporation.

From $\Delta W = P(V_2 - V_1)$

External work done = $1.01 \times 10^5 \times 2.5 = 2.53 \times 10^5$J

(ii) Explain what happens to the rest of the energy.

The difference in energy is the increase in internal energy of water molecules.

i.e. $3.39 \times 10^6 - 2.53 \times 10^5 = 3.24 \times 10^6$J = increase in internal energy of water molecules.

Molar Heat capacities

Definition

Molar heat capacity of a substance is the amount of heat required to raise the temperature of one mole of it by 1K. It is expressed in $Jmol^{-1}K^{-1}$.

Molar heat capacity at constant volume, c_v

Molar heat capacity at constant volume of a substance is the amount of heat required to raise the temperature of one mole of it by 1K at constant volume. It is expressed in $Jmol^{-1}K^{-1}$.

Molar heat capacity at constant pressure, c_p

Molar heat capacity at constant pressure of a substance is the amount of heat required to raise the temperature of one mole of it by 1K at constant pressure. It is expressed in $Jmol^{-1}K^{-1}$.

The relationship between the principal molar heat capacities C_p and C_v for an deal gas.

From dQ = dU + dW…………… (i)
But $dQ = C_pdT$, $dU = C_vdT$ and dW= PdV = RdT
Substituting in (i)

$C_pdT = C_vdT + RdT$
$\therefore C_p - C_v = R$
Where R is the universal gas constant per unit mass.

Example 23

The temperature of 1mole of helium gas at a pressure of 1.0×10^5Pa increases from 20^0C to 100^0C when the gas is compressed adiabatically.

Find the final pressure of the gas. (Take ${}^{C_p}/_{C_v}$ =γ =1.67) (04 marks)

$$P_1V_1^\gamma = P_2V_2^\gamma$$
$$but\ V = \frac{nRT}{P} => \frac{P_1T_1^\gamma}{P_1} = \frac{P_2T_2^\gamma}{P_2}$$
$$=> \frac{T_1^\gamma}{P_1^{\gamma-1}} = \frac{T_2^\gamma}{P_2^{\gamma-1}}$$
$$\frac{(293)^{1.67}}{(1.0\ x\ 10^5)^{0.67}} = \frac{(373)^{1.67}}{(P)^{0.67}}$$
$$P = 1.87\ x\ 10^5 Pa$$

Example 24

Nitrogen gas is trapped in a container by a movable piston. If the temperature of the gas is raised from 0^0C to 50^0C at constant pressure of 4.0×10^5Pa and the total heat added is 3.0×10^4J, calculate the work done by the gas. [The molar heat capacity of nitrogen at constant pressure is $29.1Jmol^{-1}K^{-1}$, C_p/C_v= 1.4]

ΔQ = ΔU + Δw …………………… (i)
$Cv = \frac{C_p}{1.4} = \frac{29.1}{1.4} = 20.79Jmol^{-1}$
$\Delta Q = nC_p\Delta T$

n = $\frac{\Delta Q}{C_p \Delta T} = \frac{3 \times 10^4}{29.1 \times 50} = 20.62$

From equation (i)

3 x 10^4 = 20.62 x 20.79 (50-0) + Δw

Δw = 8.57 x 10^3J

Example 25

Ten moles of a gas, initially at 27^0C are heated at constant pressure of 1.01 x 10^5Pa and volume increased from 0.25m^3 to 0.375m^3. Calculate the increase in internal energy.

[Assume C_p = 28.5$Jmol^{-1}K^{-1}$] (06marks)

T_1 = 27^0C = 300K

Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{0.250}{300} = \frac{0.375}{T_2}$; T_2 = 450K

$\Delta T = 450 - 300 = 150K$

ΔQ = ΔU + Δw

$nC_p\Delta T = nC_v\Delta T + nR\Delta T$

$nC_v\Delta T = nC_p\Delta T - nR\Delta T$

= 10 x 28.5 x 150 – 10 x 8.31 x 150

= 3.03 x 10^4J

Example 26

An ideal gas at 18^0C is compressed adiabatically until the volume is halved. Calculate the final temperature of the gas. [Assume specific heat capacities of the gas at constant pressure and volume are 2100$Jkg^{-1}K^{-1}$ and 1500$Jkg^{-1}K^{-1}$] respectively. (04marks)

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\gamma = \frac{2100}{1500} = 1.40$

⇨ $291(V)^{1.4-1} = T_2\left(\frac{V}{2}\right)^{1.4-1}$

T_2 = 384K

Dalton's Law

Dalton's law states that the total pressure of a mixture of gases which do not interact chemically is equal to the sum of partial pressures of the individual gases.

Definition

Partial pressure is the pressure of an individual gas in a mixture or partial pressure of a gas in a mixture is the pressure which it would exert if it were allowed to occupy the volume of the mixture at the same temperature as the mixture

The kinetic theory of matter.

- Gases are composed of molecules which are in continuous random motion.
- The molecules collide elastically with one another and also with the walls of the container.
- The pressure of a gas is due to the molecules bombarding the walls of its container. Whenever a molecule bounces off a wall, its momentum at right-angles to the wall is reversed; the force

which it exerts on the wall is equal to the rate of change of its momentum. The average force exerted by the gas on the whole of its container is the average rate at which the momentum of its molecules is changed by collision with the walls.

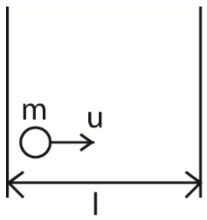
Calculation of pressure

To find the pressure of the gas we must find this force, and then divide it by the area of the walls.

Assumptions

(a) The attraction between the molecules is negligible.
(b) The volume of the molecules is negligible compared with the volume occupied by the gas.
(c) The molecules are like perfectly elastic spheres.
(d) The duration of a collision is negligible compared with the time between collisions.

Consider a molecule of mass, m, moving in a cube of length, l and velocity, u.



Change in momentum = mu - $^-$mu = 2mu

Rate of change of momentum = $\frac{2mu}{t}$

Time, t, between collision = $\frac{2l}{u}$

$F_1 = 2mu \div \frac{2l}{u} = \frac{mu^2}{l}$

For N molecules, force on the wall,

F = $\frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \cdots + \frac{mu_N^2}{l}$

Pressure, P = $\frac{F}{A} = \frac{m}{l^3}(u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2)$ since A = l^2

$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2}{N}$

$N\overline{u^2} = u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2$

$\therefore P = \frac{Nm\overline{u^2}}{l^3} = \rho\overline{u^2};\ since\ \rho = \frac{Nm}{l^3}$

$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$ and $\overline{u^2} = \overline{v^2} + \overline{w^2}$

$\therefore \overline{c^2} = 3\overline{u^2}$ => $\overline{u^2} = \frac{1}{3}\overline{c^2}$

$\therefore P = \frac{1}{3}\rho\overline{c^2}$

Definition

Mean square speed is the average square speed of the gas molecules at a particular temperature, T

Root-mean-square speed, $\sqrt{\overline{c^2}}$ is the square root of mean of square velocities of gas molecules.

Since $P = \frac{1}{3}\rho\overline{c^2}$

$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}}$ ms^{-1}

From PV = nRT and ρ = $\frac{Nm}{V}$

P = $\frac{N}{3V}m\overline{c^2}$

PV = $\frac{N}{3}m\overline{c^2}$

⇨ $\frac{N}{3}m\overline{c^2} = RT$

This shows that **mean square speed s proportional to temperature.**

Also

RT = $\frac{N}{3}m\overline{c^2} = \frac{2N}{3}\left(\frac{1}{2}m\overline{c^2}\right)$

$\frac{1}{2}m\overline{c^2} = \frac{3R}{2N}T$

$= \frac{3}{2}kT$

Where k = $\frac{R}{N}$ is Boltzmann's constant numerically equal to 1.38 x 10^{-2}JK^{-1}.

Therefore, the average kinetic energy of translation of the random motion of the molecule of a gas is proportion to the kinetic energy.

Example 27

If the mass of 1mole of hydrogen is 2.0g and this occupies a volume of 0.022m3 at 273K and pressure of 105Nm-2. Calculate the r.m.s speed of hydrogen at 546K.

$P = \frac{1}{3}\rho\overline{c^2}$

ρ = $\frac{M}{V} = \frac{2 \, x \, 10^{-3}}{0.022} = 0.0909$kgm^{-3}

$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \, x \, 10^5}{0.0909}}$ = 1.8167 x 10^{3}ms^{-1}

Let r.m.s speed at 546K be $\sqrt{\overline{c_1^2}}$

$\sqrt{\frac{\overline{c^2}}{\overline{c_1^2}}} = \sqrt{\frac{T}{T_1}}$

$\sqrt{\overline{c_1^2}}$ = $\sqrt{\frac{T_1}{T}} = \sqrt{\overline{c^2}}$ = $\sqrt{\frac{546}{273} \, x \, 1.8167 \, x \, 10^3}$ = 2569.2ms^{-1}

Example 28

Derive the expression P = $\frac{1}{3}\rho\overline{c^2}$ for the pressure, P, of an ideal gas of density ρ and mean square speed, $\overline{c^2}$. State any assumptions made (07marks)

Example 29

Derivation of Dalton's law from kinetic theory expression, p = $\frac{1}{3}\rho\overline{c^2}$, where ρ is the pressure of a gas of density ρ and mean square speed c^2

P = $\frac{1}{3}N\frac{m}{V}\overline{c^2}$ = $\frac{2}{3}N\left(\frac{1}{2}m\overline{c^2}\right)$

For gas 1, P_1V_1 = $\frac{2}{3}N_1\left(\frac{1}{2}m_1\overline{c_1^2}\right)$

=> $N_1 = \frac{3}{2}P_1V_1.\frac{1}{K_1}$

Similarly for gas 2

$N_2 = \frac{3}{2}P_2V_2.\frac{1}{K_2}$

For a mixture of gases, N = $\frac{3}{2}PV.\frac{1}{K}$; but N = $N_1 + N_2$

$\frac{3}{2}PV.\frac{1}{K} = \frac{3}{2}P_1V_1.\frac{1}{K_1} + \frac{3}{2}P_2V_2.\frac{1}{K_2}$

Since temperature is constant, $K_1 = K_2 = K$

- $PV = P_1V_1 + P_2V_2$
- But $V = V_1 = V_2$
- $\therefore P = P_1 + P_2$

Example 30

Explain why the pressure of a fixed mass of a gas rises if its temperature is increased. (02marks)

When the temperature of a fixed mass of a gas is increased, at constant volume, the velocities and kinetic energy of molecules is increased. They bombard the walls of the container more frequently with increased force. This increases pressure since pressure is proportional to force.

Example 31

Explain the following observations using the kinetic theory.

(i) A gas fills any container in which is it placed and exerts pressure on its walls. (03marks)

A gas contains molecules with negligible intermolecular forces and free to move in all directions. As they move, they collide with each other and with the walls of the container. The unrestricted movements make them to fill the available space and collisions with the walls contributes to the pressure exerted on the wall.

(ii) The pressure of a fixed mass of a gas rises when temperature is increased at constant volume. (02 marks)

When the temperature of a gas increases, the kinetic energy of the gas molecules increases. This increases the frequency and force of collision against the wall leading to increase in pressure.

Example 32

Explain why the pressure of a fixed mass of a gas in a closed container increases when the temperature of the container is raised. (02marks)

When the temperature of the container increases, the average velocity of the molecules increases. So the number and force of collisions with the walls of the container per second increases. Consequently the momentum change per second increases as they bombard the walls. This leads to increase in the impulsive force exerted on the walls causing increase in pressure

Graham's law

States that the rate of diffusion of a gas is inversely proportional to the square root of its density.

From $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$ where k is a universal constant

At the same Kelvin temperature T, the mean kinetic energies of the molecules of different gases are equal.

If subscript 1 and 2 denote gases 1 and 2 respectively

$$\frac{1}{2}m_1\overline{c_1^2} = \frac{1}{2}m_2\overline{c_2^2}$$

$$\frac{\overline{c_1^2}}{\overline{c_2^2}} = \frac{m_2}{m_1}$$

At a given temperature and pressure, the density of a gas, ρ, is proportional to the mass of its molecule, m, since equal volumes contain equal number of molecules

Therefore $\frac{m_1}{m_2} = \frac{\rho_2}{\rho_1}$

Then $\frac{\overline{c_1^2}}{\overline{c_2^2}} = \frac{\rho_2}{\rho_1}$

Hence, $\frac{\sqrt{\overline{c_1^2}}}{\sqrt{\overline{c_2^2}}} = \frac{\sqrt{\rho_2}}{\sqrt{\rho_1}}$

The equation

(i) shows that the average molecular speeds are inversely proportional to the square roots of the densities of the gases.

(ii) explains why the rates of diffusion-which depend on the molecular speeds-are also inversely proportional to the square roots of the densities.

Real gases

As opposed to ideal gases, in real gas;

- The volume of the molecules may not be negligible in relation to the volume V occupied by the gas.
- The attractive forces between the molecules may not be negligible.

Real gas equation

To account for molecular volume and intermolecular forces J.H. Vander Waal proposed the following equation

$$(P+\frac{a}{V^2})(V-b) = RT$$

For 1mole of a gas

$\frac{a}{V^2}$ corrects deficit in pressure due to intermolecular attractions of gas molecules and

b, called the co-volume accounts for the finite volume of molecules themselves

NB: (i) A real gas obeys ideal gas equation above the critical temperature

Differences between ideal and real gases

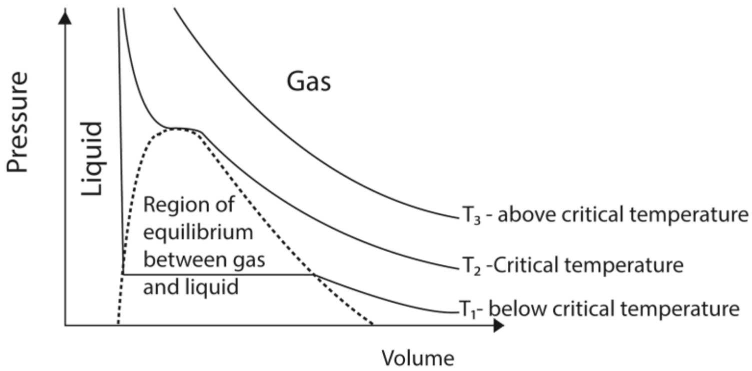
(i) Ideal gases obey Boyle's law while real gases do not.
(ii) The volume of the molecules of ideal gases are negligible compared to the container while for the real gases the volume is not negligible.
(iii) Ideal gases have negligible intermolecular forces while the intermolecular forces of real gases are not negligible
(iv) The velocity of ideal gas molecules is constant in between collision while real gases molecules do not have constant velocity due to intermolecular forces.

Vapours

A gas is the gaseous state of a substance above its critical temperature, T_C.

A vapour is the gaseous state of a substance below its critical temperature.

A graph of a real gas below and above the critical temperature



- Above the critical temperature a gas obeys Boyle's law.

- Below the critical temperature a gas exist as unsaturated vapour at low pressure when the pressure is increase it condenses until all the gas is turned into a liquid.

A critical temperature is a temperature above which a gas cannot be turned into a liquid by compression.

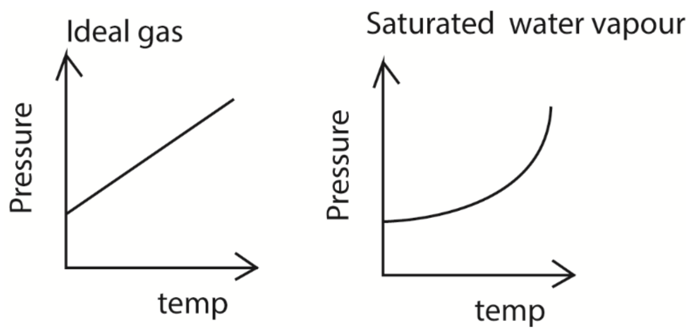
Saturated and unsaturated vapour

Unsaturated vapour is a vapour that is not in dynamic equilibrium with its own liquid while saturated vapour is a vapour that is in dynamic equilibrium with its own liquid

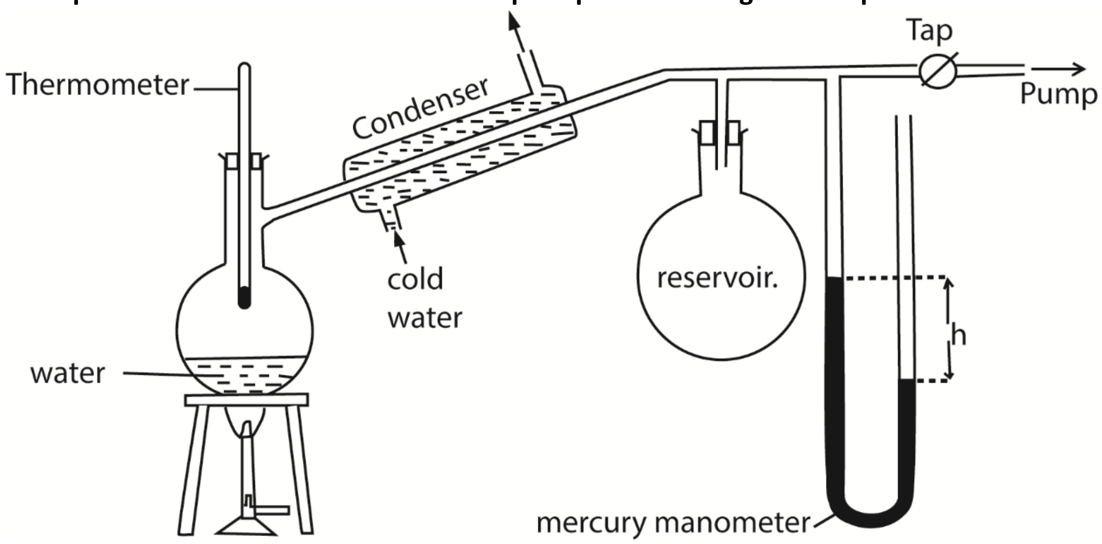
Differences between saturated and unsaturated vapours.

- A saturated vapour pressure is one which is in dynamic equilibrium with its own liquid and an unsaturated vapour is one that is not in dynamic equilibrium with its own liquid.
- A saturate vapour pressure does not obey gas laws whereas unsaturated vapour approximately obey gas laws.

Sketches of graphs of pressure against temperature for an ideal gas and for saturated water vapour originally at 0^0C (03marks)



An experiment to determine saturated vapour pressure at a given temperature.



- The pressure of the air in R is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.

- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = $(H \pm h)$ is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and the saturated vapour pressure at a particular temperature is obtained.

Example 33

With the aid of a labelled diagram, describe an experiment to determine standard saturated vapour pressure of water. (05marks)

Example 34

Use the kinetic theory to explain the following observations

(i) Saturated vapour pressure of a liquid increases with temperature. (03marks)
If a liquid is in dynamic equilibrium with its vapour, an increase in temperature increases mean kinetic energy of the molecules and hence the rate at which molecules escape from the liquid. The density of the vapour increases implying increase in the rate of condensation until dynamic equilibrium is restored. There are now more molecules in the vapour phase than previously that are moving faster and hence higher pressure.

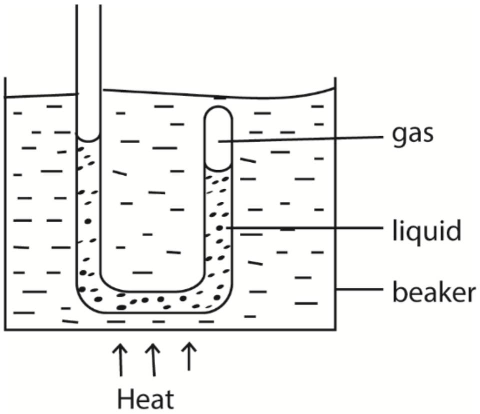
(ii) (ii) Saturated vapour pressure is not affected by decrease in volume at constant pressure. (03marks)
A decrease in volume leads to a momentary increase in vapour density. Consequently, the rate of condensation increases while the rate of evaporation rate is constant. When the vapour density reduces, the condensation rate also reduces. So the dynamic equilibrium is restored to the initial value.

Example 35

Two similar cylinders P and Q contain different gases at the same pressure. When gas is released from P the pressure remains constant for some time before it starts dropping. When gas is released from Q the pressure continuously drops. Explain the observation above. (05marks)

- The gas in P is in form of a saturated vapour; that is, in dynamic equilibrium with a liquid. As the gas is released, more liquid turns into a gas to restore pressure until the gas becomes unsaturated and the pressure begins to drop as the moles of the gas decrease
- The gas in Q is unsaturated, and thus pressure reduces as the moles of the gas reduces up on release.

Experiment to show that a liquid boils off when its saturated vapour pressure equals the external pressure



- Air is trapped in the closed limb of the tube by water column.
- The tube is heated in water bath.
- When the water bath begins to boil, the water in the tube comes to the same level in each limb.
- This shows that the vapor pressure in closed limb is equal to external pressure.

Evaporation

During evaporation, a liquid changes to vapour low the boiling point.

Reasons why evaporation causes cooling

During evaporation liquid molecules with high kinetic energy escape from inter molecular attraction in the liquid leaving molecules of low kinetic energy behind. Since temperature s proportional average kinetic energy of molecules, the liquid cools.

Factors affecting the rate of evaporation

(i) Surface area. Evaporation increases with surface area of a liquid due to increase in the number of exposed molecules.

(ii) Temperature: increase in temperature increases the rate of evaporation because it increases the average kinetic energy of the molecules.

(iii) Drought /wind blowing over the surface. Wind removes the saturated air layer from the surface of the liquid thereby increasing the rate of evaporation.

Example 36

Explain the occurrence of land and sea breeze. (04marks)

During day, the land is heated to a high temperature than the sea. Hot air expands and rises from land. A stream of cool air from the sea blows towards the land to replace the uprising air, hence sea breeze occurs.

At night the land cools faster because it is no longer heated by the sun. The sea retains the warmth because it is heated deeply. Warm less dense air rises from the sea surface and air from land blows to replace it leading to land breeze.

Example 37

A horizontal tube of uniform bore, closed at one end, has some air trapped by a small quantity of water. The length of the enclosed air column is 20cm at 12^0C.

Find stating any assumptions made, the length of air column when the temperature is raised to 38^0C.

[S.V.P of water at 12^0C and 38^0C are 10.5mmHg and 49.5mmHg respectively. Atmospheric pressure = 75cmHG] (05marks)

T_1 = 273 + 12 = 285K, T_2 = 273 + 38 = 311K; P_1 = 750 – 10.5 = 739.5mmHg, P_2 = 750-49.5 = 700.5mmHg

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{739.5\ x\ 20A}{285} = \frac{700.5\ x\ hA}{311}$$

h = 23.04cm

Assumption: the tube does not expand when the temperature increases.

Example 38

When hydrogen gas is collected over water, the pressure in the tube at 15^0C and 75^0C are 65.5cm and 105.6cm of mercury respectively. If the saturated vapour pressure at 15^0C is 1.42cm of mercury, find its value at 75^0C (04marks)

P_1= 65.5 – 1.42 = 62.08 and T_1 = 273 + 15 = 288K

P_2 = 105.6 – P, and T_2 = 273 + 75 = 348K

From $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

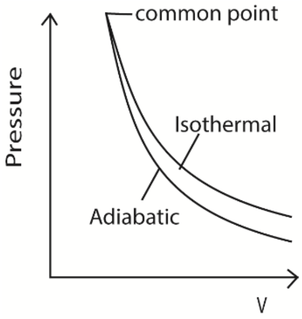
P_2 = $\left(\frac{P_1}{T_1}\right) T_2$ =105.6 - $\frac{64.08}{288}\ x\ 348$ = 28.12cmHg

Isothermal and adiabatic changes

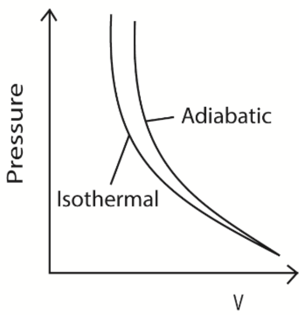
Isothermal expansion takes place at constant temperature.

Adiabatic expansion takes place at constant heat.

Sketch graphs of pressure versus volume for fixed mass of a gas undergoing isothermal and adiabatic changes.



Or



Condition necessary for realization of an isothermal change

(i) The gas must be held in thin-walled and highly conducting vessel
(ii) The process must take place slowly so that heat pass into the gas to maintain constant temperature.
(iii) The gas vessel must be surrounded by a constant temperature bath.

Conditions for adiabatic change

(i) The gas must be held in a thick walled and poorly conducting vessel
(ii) The process must be carried out rapidly to minimize heat linkage through the walls.

Relationship between volume, pressure and temperature

(i) For reversible isothermal change
PV = nRT where n is the number of mole of gas, R = gas constant
(ii) For reversible adiabatic change
Pv^{γ} = constant
$TV^{\gamma-1}$ = constant

Example 39

Ten moles of a gas, initially at 27^{0}C are heated at constant pressure of 1.01 x 10^5Pa and volume increased from 0.25m^3 to 0.375m^3. Calculate the increase in internal energy. [Assume C_p = 28.5$Jmol^{-1}K^{-1}$] (06marks)

T_1 = 27^{0}C = 300K

Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{0.250}{300} = \frac{0.375}{T_2}$; T_2 = 450K

$\Delta T = 450 - 300 = 150K$

ΔQ = ΔU + Δw

$nC_p\Delta T = nC_v\Delta T + nR\Delta T$

$nC_v\Delta T = nC_p\Delta T - nR\Delta T$

= 10 x 28.5 x 150 – 10 x 8.31 x 150

= 3.03 x 10^4J

Example 40

An ideal gas at 18^{0}C is compressed adiabatically until the volume is halved. Calculate the final temperature of the gas. [Assume specific heat capacities of the gas at constant pressure and volume are 2100$Jkg^{-1}K^{-1}$ and 1500$Jkg^{-1}K^{-1}$] respectively. (04marks)

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\gamma = \frac{2100}{1500} = 1.40$

⇨ $291(V)^{1.4-1} = T_2\left(\frac{V}{2}\right)^{1.4-1}$

T_2 = 384K

Example 41

State the first law of thermodynamics and use it to distinguish between Isothermal and adiabatic changes in a gas. (05marks)

$$\Delta Q = \Delta U + \Delta W = nC_v\Delta T + \Delta W$$

During isothermal expansion, $\Delta T = 0$. Therefore all the energy supplied is equal to the work done by the gas during expansion.

In adiabatic expansion, no heat enters or leaves the gas. Therefore $\Delta Q = 0$ and $\Delta U = -\Delta W$.

In adiabatic expansion, work s done at the expense of its internal energy. Therefore the gas cools.

Example 42

(a) (i) What is meant by isothermal process and adiabatic process? (02marks)

Isothermal process is the expansion or compression of a gas at constant temperature.

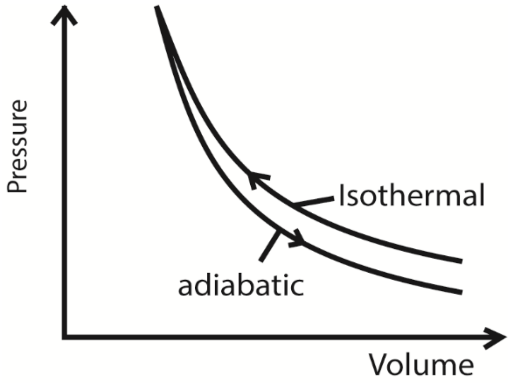
Adiabatic process is the expansion or compression of a gas where there is no heat loss or gain into the gas.

(ii) Explain why adiabatic expansion of a gas causes cooling (03marks)

During an adiabatic expansion of a gas, no heat is supplied to the gas. Molecules strike the receding piston and bounce off with reduced velocities and hence lower kinetic energy. Since kinetic energy is proportional to temperature, the gas cools during the expansion

(b) A gas at a temperature of 17^0C and pressure 1.0×10^5Pa compressed isothermally to half its original volume. It is then allowed to expand adiabatically to its original volume

(i) Sketch a P-V curve the above process (02marks)



(ii) If the specific heat capacity at constant pressure is $2100Jmol^{-1}K^{-1}$ and at constant volume is $1500Jmol^{-1}K^{-1}$, find the final temperature of the gas (04marks)

$\gamma = \frac{2100}{1500} = 1.4$

$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

$290\left(\frac{V}{2}\right)^{0.4} = T_3V^{0.4}$

$T_3 = 219.8K$

Heat transfer

Conduction

Conduction is the transfer of heat from a region of high temperature to that of low temperature without a resultant movement of the molecules of conducting material.

Mechanisms of heat conduction

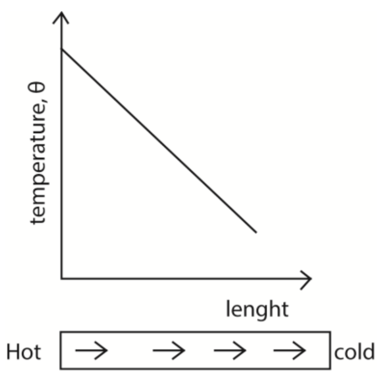
(a) Metals
- The atoms of metals consist of free mobile electrons; when one end of a metal is heated, these free electrons travel at high speed and collide with other electrons and atoms. In this way heat is transferred quickly from a hot end to a cold end.
- Secondly when one end of a metal is heated atoms vibrate with high frequency and amplitude; collide with other atoms to which they give heat. Those atoms that receive heat also vibrate with high frequency and amplitude, collide and transfer their heat. In this way heat energy is transferred from one part of the metal to another.

(b) Non-metals
- In non-metal heat is transferred by interatomic vibrations.
- When one end of a metal is heated atoms vibrate with high frequency and amplitude; collide with other atoms to which they give heat. Those atoms that receive heat also vibrate with high frequency and amplitude, collide and transfer their heat. In this way heat energy is transferred from one part of the metal to another

(c) Heat transfer in gases
Heat energy in gases is transferred by molecular collisions between hot and cold molecules. When a gas is heated, the fast moving molecules collide and pass on their kinetic energy to cold slower molecules.

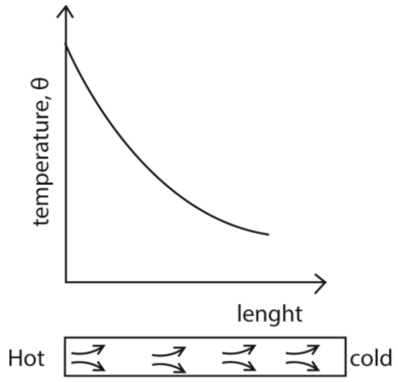
Temperature distribution along a conductor

(i) **lagged metal**



The rate of heat flow along the bar is constant since heat loss is negligible.

(ii) **Unlagged or exposed to the surrounding**

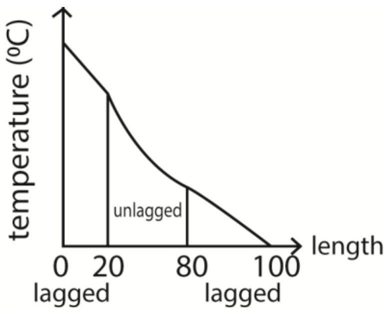


The rate of heat flow decrease with length due to heat losses

Example 43

The two ends of a metal bar of length 1.0m are perfectly lagged up to 20cm from either end. The ends of the bar maintained at 100^0C and 0^0C respectively.

(i) Sketch a graph of temperature versus distance of a bar. (02marks)



(ii)Explain the features of the graph in (b)(i)(03marks)

- In lagged portions there is constant heat flow because there I no heat loss to the surroundings
- In unlagged portion heat flow is not uniform due to heat loss to the environment.

Thermal conductivity.

Thermal conductivity is the rate of heat transfer per unit cross section area per unit temperature gradient

Factors affecting the rate of heat flow

- Temperature gradient
- Cross-section area

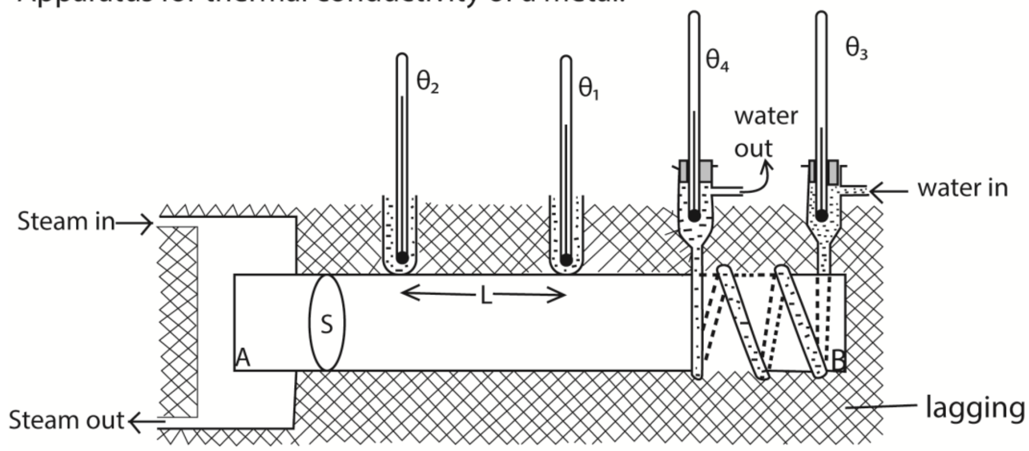
i.e. $\frac{Q}{t} = kA\left(\frac{\theta_2 - \theta_1}{L}\right)$

Measurement of thermal conductivity of a good conductor (e.g. metal)

Conditions

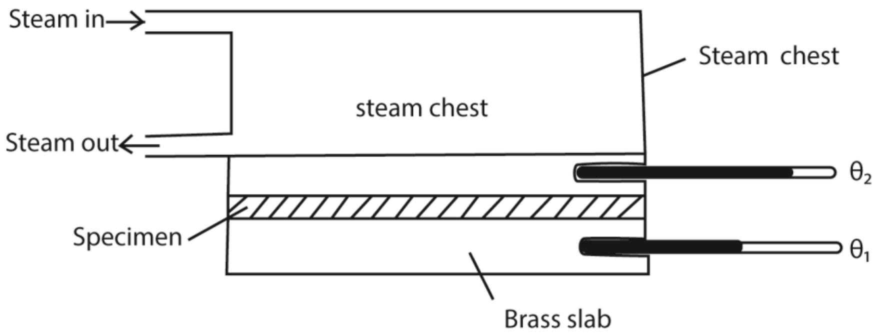
- Heat must flow through the specimen at measurable rate
- The temperature gradient along the specimen must be measurably steep. i.e. the specimen bar must be longer than it diameter.

Apparatus for thermal conductivity of a metal.



- Specimen bar AB of mean diameter, d, is heated by steam at end A and cooled by water at end B as shown above
- The lagging ensure a constant rate of heat flow
- The setup is left to run for some time until steady temperatures θ_1, θ_2, θ_3 and θ_4 are obtained.
- The rate water flow m kgs^{-1} is measured using a cylinder and stop clock.
- Cross section area A = $\frac{\pi d^2}{4}$
- The rate of heat flow is given by
  $\frac{Q}{t} = kA\left(\frac{\theta_2 - \theta_1}{L}\right) = mc_w(\theta_4 - \theta_3)$
  where k = thermal conductivity of the metal and c_w is specific heat capacity of water

Measurement of thermal conductivity of a bad conductor (e.g. glass, cork)



- Glass s cut in form of a thin disc of cross section area, A and thickness, x.
- The disc is sandwiched between a steam chest and brass slab of mass, m and specific heat capacity, c.
- Steam is passed through the chest until the thermometers register steady temperatures, θ_1 and θ_2.

- Then, $\frac{\theta}{t} = kA\left(\frac{\theta_2 - \theta_1}{x}\right)$
- The glass disc is removed and brass slab is heated directly by steam chest, until its temperature is about 10^0C above θ_1.
- Steam chest is removes and the top of the glass slab is covered by the glass disc.
- The temperature of the slab is recorded at suitable time interval until its temperature is about 10^0C below θ_1.
- A graph of temperature against time is plotted and its slope s determined at θ_1

$$\frac{\theta}{t} = mcs$$

$$\therefore\ kA\left(\frac{\theta_2 - \theta_1}{x}\right) = mcs$$

$$k = \frac{mcsx}{A(\theta_2 - \theta_1)} \text{ but } A = \frac{\pi D^2}{4}$$

$$\therefore k = \frac{4mcsx}{\pi D^2(\theta_2 - \theta_1)}$$

Precautions

- Sample in a thin disc
- Faces of the disc highly polished to ensure tight uniform contacts
- A thin layer of grease is smeared on faces for good thermal contact.

Example 44

A cylindrical iron vessel with a base of diameter 15cm and thickness 0.30cm has its base coated with a thin film of soot of thickness 0.10cm. It is then filled with water at 100^0C and placed on a large block of ice at 0^0C. Calculate the initial rate at which the ice will melt (06marks) (thermal conductivity of soot=$0.12Wm^{-1}K^{-1}$, Thermal conductivity of iron, k = $75Wm^{-1}K^{-1}$)

$$\frac{Q}{t} = kA\left(\frac{\theta_2 - \theta_1}{x}\right) = k_1A\left(\frac{\theta_2 - \theta_1}{x}\right) = ml_f$$

where m is mass that melt per second and l_f = latent heat of fusion

$$\frac{Q}{t} = 75A\left(\frac{100 - \theta_1}{0.3\ x\ 10^{-2}}\right) = 0.12A\left(\frac{\theta_1 - 0}{0.1\ x\ 10^{-2}}\right)$$

$\theta_1 = 99.52^0C$

Also, $kA\left(\frac{\theta_2 - \theta_1}{x}\right) = ml_f$

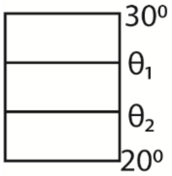
$$75\ x\ \pi\ x\ \frac{(0.15)^2}{4}\left(\frac{100 - 99.52}{0.3\ x\ 10^{-2}}\right) = m\ x\ 3.3\ x\ 10^5$$

m = $6.42 \times 10^{-4}kgs^{-1}$

Example 45

A window of height 1.0m and width 1.5m contains a double grazed unit consisting of two single glass panes, each of thickness 4.0mm separated by an air gap of 2.0mm. Calculate the rate at which heat is conducted through the window if the temperatures of external surfaces of glass are 20^0C and 30^0Crespectively.

[Thermal conductivities of glass and air are $0.72Wm^{-1}K^{-1}$ and $0.025\ Wm^{-1}K^{-1}$ respectively] (07marks)



$\frac{dQ}{dT} = \frac{kA(\theta_2 - \theta_1)}{L}$ = mc x slope

$$\frac{k_1A(30 - \theta_1)}{4\ x\ 10^{-3}} = \frac{k_2A(\theta_2 - \theta_1)}{2\ x\ 10^{-3}} = \frac{k_1A(\ \theta_1 - 20)}{4\ x\ 10^{-3}}$$

=> $\theta_1 + \theta_2 = 50$

$$\frac{0.72A(30 - \theta_1)}{4\ x\ 10^{-3}} = \frac{0.025A(\ \theta_1 - \theta_2)}{2\ x\ 10^{-3}}$$

$$\frac{0.72A(30 - \theta_1)}{4\ x\ 10^{-3}} = \frac{0.025A(\ \theta_1 - (50 - \theta_1)}{2\ x\ 10^{-3}}$$

$\theta_1 = 29.4^0C$

Hence $\frac{dQ}{dT} = \frac{0.72A(30 - 29.4)}{4\ x\ 10^{-3}}$ =162W

Example 46

Explain why heating system based on the circulation of steam are more efficient than those based on circulation of boiling water. (02marks)

A given mass of steam gives out more energy that an equal amount of water because of the specific latent heat

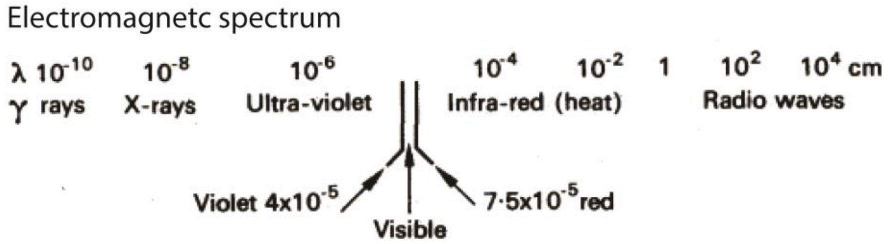
Radiation

Thermal radiation is the transfer of heat through vacuum i.e. no material medium is required for this transmission.

A black body radiation is an electromagnetic radiation emitted by a body solely due its temperature i.e. energy emitted depends on body's temperature.

Electromagnetic spectrum

Electromagnetic spectrum is the distribution of electoral magnetic radiations ranging from those of short wave length to those of longer wave length as shown below.



Infrared radiations

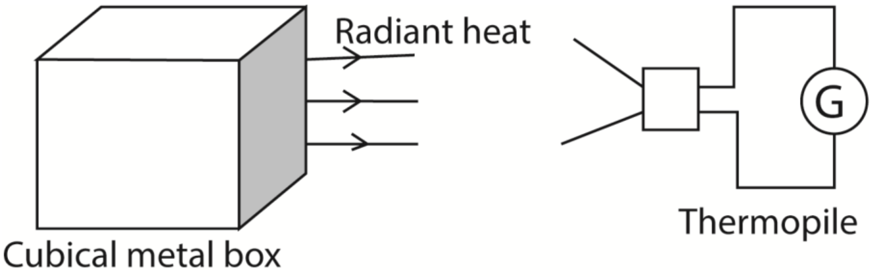
Infrared radiations are electromagnetic radiations which are converted into heat when they strike a surface.

Properties of infrared radiations.

- They travel at the speed of light.
- They are reflected and refracted like light.
- When absorbed by a body the body's temperature is raised.
- Causes photo-electric emission from surfaces like Cesium.
- Affects special types of photographic plates which enable pictures to be taken in the dark.
- They are absorbed by glass but transmitted by rock salt and quartz.

Comparison of radiation ability for different surfaces

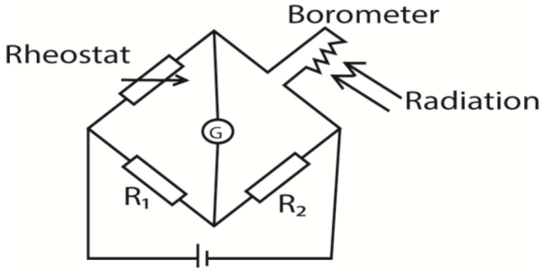
A cubical metal tank whose sides are painted; dull black, dull white and highly polished is filled with hot water and radiations from each surface are detected by a thermopile as shown below.



The galvanometer deflection is greatest when the thermopile is facing the dull black surface and least when facing a highly polished silver surface. Therefore, a polished surface is the least radiator and a black surface is the best radiator.

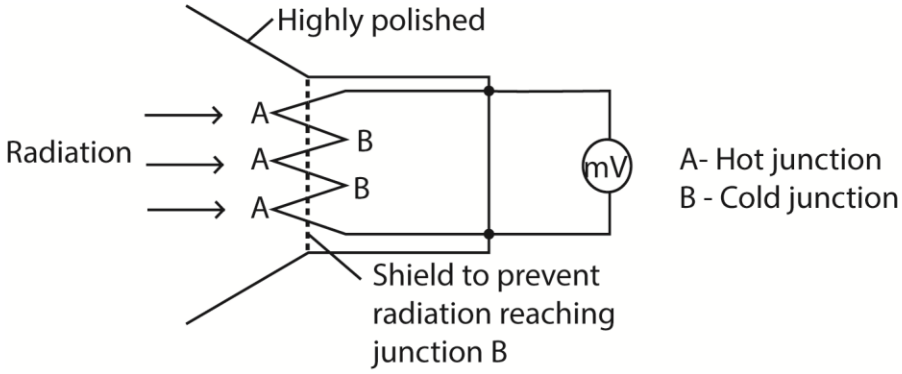
Detectors of infrared radiation

(a) Barometer



The bolometer strip is connected to Wheatstone bridge circuit above. The rheostat is adjusted until the galvanometer shows no deflection. When the radiations fall on the strip, they are absorbed and its temperature rises leading to an increase in resistance. The galvanometer deflects showing the presence of radiations.

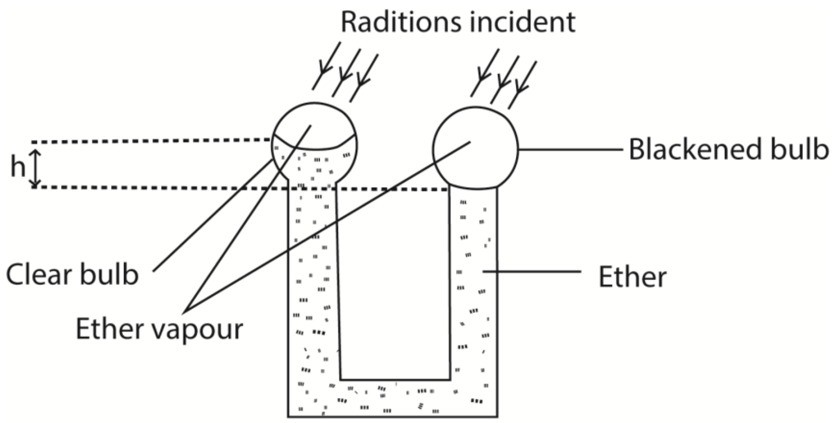
(b) a thermopile



Radiation falling on junction A is absorbed and temperature rises above that of junction B. An e.m.f is generated and is measured by millivolt meter which deflects as a result.

(c) The ether-thermo scope.

A blackened and clear bulbs are connected to a tube partly filled with ether i.e. each bulb contains mixture of air and ether vapour. When the arrangement is exposed to infrared radiations, more radiations are absorbed by the blackened bulb than those absorbed by the clear bulb. This raises the pressure inside the blackened bulb causing the ether liquid to be raised in the unblackened bulb as shown below.

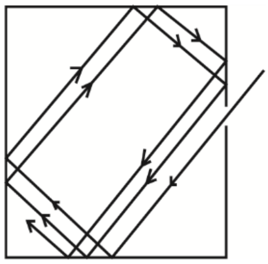


The rise h is proportional to incident radiation

Black body

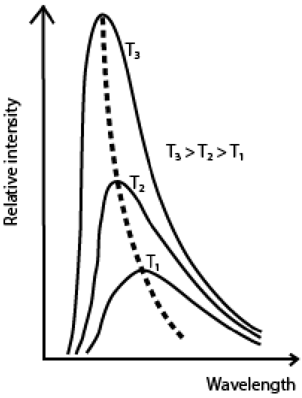
- A black body is one which absorbs all radiations incident on it and reflects or transmits none.
- The radiations emitted by a black body are called temperature radiations or black body radiations.

Approximation of black body



When radiation enters a black container through a hole, it undergoes multiple reflections. At each reflection, part of the radiation is absorbed. After several reflections, all the radiation is retained inside the container. Hence it approximates to a black body.

Distribution of black body radiation



- For every wave length, relative intensity increases as temperature is increased.
- The wavelength at which maximum intensity occur shifts to the shorter wavelength as temperature is increased.
- λ_{max} is the wavelength of radiation emitted at maximum intensity/emission of a black body at a particular temperature.

Laws of black body radiation

(a) Wien's displacement law
If λ_{max} is the wavelength of the peak of the curve for a given temperature, T then $\lambda_{max}T$ = constant; the constant is Wien's constant = 2.9×10^{-3}mK
(b) If $E\lambda_{max}$ is the height of the peak of the curve for temperature, T, then $E\lambda_{max} \propto T^5$.
(c) The curve showing the variation of E_λ with λ at a constant temperature obeys the Plank formula; $E = \frac{C_1}{\lambda^5\left(e^{\frac{c_2}{\lambda T}}-1\right)}$ where $c_1$and c_2 are constant
(d) State Stefan's law. (01mark)

Stefan's law states that the total power radiated by a black body per unit surface area is proportional to the fourth power of its absolute temperature. i.e. $\frac{P}{A} \propto T^4$

If E is the total energy radiated per second per m^2 area of a black body at temperature T, then $E = \sigma T^4$. Where σ is called Stefan's constant = $5.67 \times 10^{-8} Wm^{-2}K^{-4}$.

Example 47

The earth receives energy from the earth from the sun at the rate of $1.4 \times 10^3 Wm^{-2}$. If the ratio of the earth's orbit to the sun's radius is 216, calculate the surface temperature of the sum. (05marks)

Power radiated by the sun = $4\pi r^2\sigma T^4$

Energy intensity = $\frac{4\pi r^2\sigma T^4}{4\pi R^2}$

$\therefore \frac{4\pi r^2\sigma T^4}{4\pi R^2} = 1.4\ x\ 10^3$

$T^4 = \frac{1.4\ x\ 10^3}{5.7\ x\ 10^{-3}}\left(\frac{R}{r}\right)^2 = \frac{1.4\ x\ 10^3}{5.7\ x\ 10^{-3}}\ x\ 216^2$

$T = 5.82x\ 10^3$ K

Example 48

A spherical black body of radius 2.0cm at -73°C is suspended in an evacuated enclosure whose walls are maintained at 27°C. If the rate of exchange of thermal energy is equal to $1.85Js^{-1}$,

(i) find the value of Stefan's constant, (05marks)

$T_1 = 27 + 273 = 300K$

$T_2 = -73 + 273 = 200K$

$P = A\sigma(T_1^4 - T_2^4)$

$1.85 = 4\pi(0.02)^2\sigma(300^4 - 200^4)$

$\sigma = 5.66 \times 10^{-8} Wm^{-1}K^{-4}$

(ii) (Calculate the wavelength at which the radiation emitted by the enclosure ha maximum intensity (03mark)

$\lambda_{max}T = 2.9 \times 10^{-3}$

$\lambda_{max} = \frac{2.9\ x\ 10^{-3}}{300} = 9.7x\ 10^{-6}m$

Prevost's theory of heat exchange

When a body is in thermal equilibrium with its surrounding, its rate of emission of radiation to surrounding is equal to the rate of absorption of the radiation from the surrounding.

Example 49

A small blackened solid copper sphere of radius 2cm is placed in an evacuated enclosure whose wall are kept at 100°C. Find the rate at which energy must be supplied to the sphere to keep its temperature at 127°C. (03marks)

P =σA($T_2^4 - T_1^4$)

= σ4πr^2($T_2^4 - T_1^4$)

= 5.67 x 10^{-8} x 4π (2 x 10^{-2})^2(400^4 – 372^4)

= 1.78W

Example 50

The energy intensity received by a spherical planet from a star is 1.4 x 10^{3}Wm^{-2}. The star is of radius 7.0 x 10^{5}km and is 14.0 x 10^{7}km from the planet.

(i) Calculate the surface temperature of the star. (04marks)

Incident energy per second (power) on a unit of planet = $\frac{4\pi r_s^2 \sigma T^4}{4\pi R^2}$

$=\left(\frac{r_s}{R}\right)^2 \sigma T^4$

Where r_s and R are the radii of the star and the distance of the star from the planet respectively.

1.4 x 10^3 = $=\left(\frac{r_s}{R}\right)^2 \sigma T^4$

T = $\left[\frac{1.4 \ x \ 10^3}{\sigma} x \left(\frac{R}{r_s}\right)^2\right]^{\frac{1}{4}} = \left[\frac{1.4 \ x \ 10^3}{5.67 \ x \ 10^{-8}} x \left(\frac{14 \ x \ 10^{10}}{7 \ x \ 10^8}\right)^2\right]^{\frac{1}{4}}$ = 5605.98K

(ii) State any assumptions you have made in (c)(i) above (01marks)

- The star is spherical
- The star radiates as a black body
- There is no heat loss to the sphere

Greenhouse effect and global warming

- When short wavelength infrared radiation from the sun pass through the water vapour and carbon dioxide in lower layers of the atmosphere, the radiation is absorbed by the earth warming it up.
- The earth re-emits this radiation (infrared) as black body radiation of long wavelength (because of low temperature) and therefore it is trapped by the water vapour and carbon dioxide in the earth's atmosphere.
- Since the radiation is prevented from escaping from the earth's atmosphere, it causes global warming.

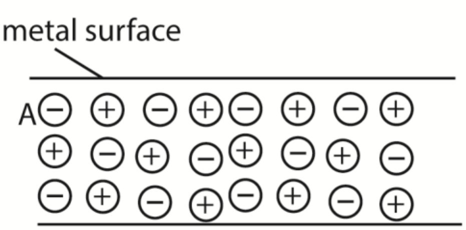
End

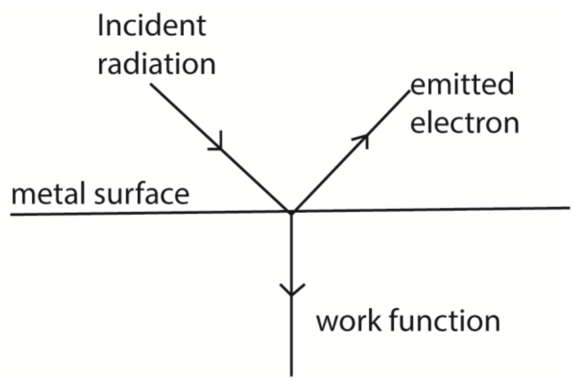
Compiled by Dr. Bbosa Science

Modern Physics

Photoelectric effect

In metals, atoms exists as positive ions in a sea of electrons. An electron near the surface of the metal, say A, experiences an attractive inward force from the positive charges below it.





For such an electron to escape from the metal surface, a specific amount of work has to be done to overcome the forces which are inward.

Definitions

Photoelectric emission: this is the liberation of an electron from a metal surface by use of light of a suitable frequency.

Thermionic emission: this the liberation of an electron from a metal surface by application of heat. N.B - The light (radiation) supplies the electrons with an amount of energy equal or exceeding the energy that binds them to the surface.

- The liberated electrons are called photo electrons.

- Surfaces which are able to undergo electric emission are said to be photo emissive e.g. K, Na, Ca, etc. generally group I elements. These have low ionization energy or low work function

- The occurrence of photoelectric effect can be demonstrated by using a gold leaf electroscope and a suitable metal e.g. zinc.

Laws of photoelectric emission

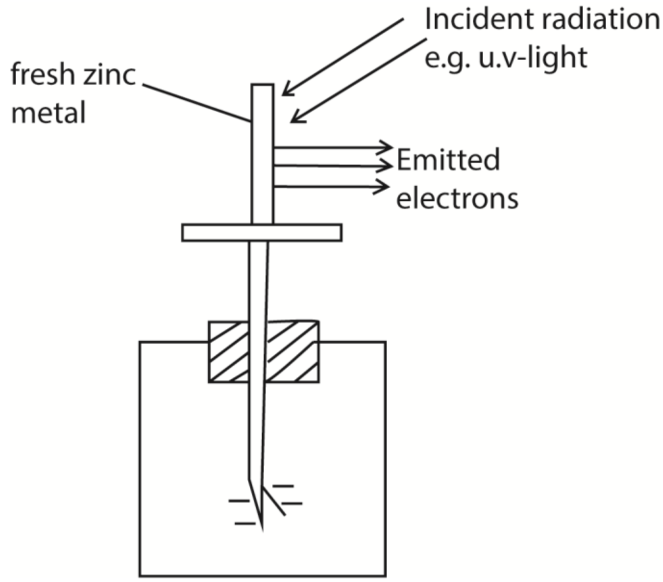
The laws (characteristics or features) are just a summary of experimental results on photoelectric effect.

1. The time lag between irradiation of the metal surface and emission of the electrons by the metal surface is negligible.
2. For a given metal, surface there is a minimum value of frequency of radiation called threshold frequency (f_0) below which no photo electrons are emitted from the metal however intense the incident radiation may be.
3. The number of photoelectrons emitted from the surface per second is directly proportional to the intensity of incident radiation for a particular incident frequency
4. The K.E of the photoelectrons emitted is independent of the intensity of the incident radiation but depends only on its frequency

A simple experiment to demonstrate Photo electric effect

(i) A freshly cleaned Zinc plate is connected to the cap of a negatively charged gold leaf electroscope.

(ii) Ultra violet radiations are allowed to fall on the zinc plate



Observations

- The leaf of the electroscope gradually falls
- This shows that both the zinc plate and the electroscope have lost charges.
- The lost charges are found to be electrons, hence photoelectric effect has occurred.

Note: If a positively charged electroscope is used instead, there is no observable change in the divergence of the leaf because the emitted electrons are immediately attracted back by the positive charges on the cap of the electroscope hence restoring the charges.

Planks Quantum theory

States that the energy /radiation emitted or absorbed is discrete or in packets called quanta.

That's, we can have integral values such as 1, 2, 3 … n, but not fractional amount of energy

The energy E, contained in a quantum of radiation is proportional to the frequency f, of the radiation i.e. $E \propto f$ or $E = hf$ where h = Planks constant (6.626×10^{-34}Js)

Dimensions of h

$$h = \frac{energy}{frequency} = \frac{force\ x\ distance}{frequency}$$

$$=> [h] = \frac{[force][distance]}{[frequency]} = \frac{MLT^{-2} x\ L}{T^{-1}} = ML^2T^{-1}$$

For an electromagnetic radiation of wavelength, λ; we have $c = \lambda f$

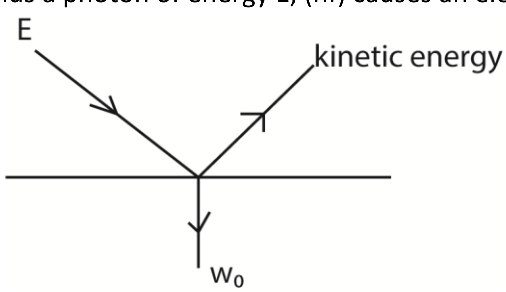
⇨ $E = \frac{hc}{\lambda}$

Thus $E \propto f$ and $E \propto \frac{1}{\lambda}$

The quantum theory of photoelectric effect

- Light energy is emitted and absorbed in discrete packets of energy called photons.
- Each photon carries (or delivers) a packet of energy or quanta given by hf. Where f is the frequency of the light/radiation and h is Plank's constant.
- It is the photon that knocks off electrons from the metal surface.
- When the photon (of energy hf) collides with an electron, it is either
  a) Reflected with no change in its energy or
  b) Absorbed by the electron and the photon gives up all its energy to that single electron without sharing with other electrons
- To liberate/eject an electron from a particular metal surface, a quantity of energy called work function w_o (which is characteristic of the metal) has to be supplied by the incident radiation

  Thus a photon of energy E, (hf) causes an electron to be emitted from the metal surface



If the energy E, (hf) is greater than the work function (w_o) of the metal, the excess energy ($hf - w_o$) is absorbed as the K.E of the emitted electron or photoelectron

i.e. $hf - w_0 = \frac{1}{2} mv^2$ where v is the velocity of emitted electron

or $hf = w_0 + \frac{1}{2} mv^2$; also called Einstein photo electric equation

The emitted electron escapes with a velocity having any value up to a maximum. The value of maximum velocity depends on:

i) The work function, w_0 of the metal and,
ii) The frequency f of the incident radiation

From, $hf - w_0 = \frac{1}{2} mv^2$

- hf = energy of incident radiation of frequency, f
- w0 = work function of the metal. It is defined as the minimum amount of energy required to release an electron from a metal surface.
- $\frac{1}{2} mv^2$ = the maximum kinetic energy of the emitted electron
- If a photon has just enough energy to liberate the electron, the emitted electron gains no kinetic energy and therefore floats on the surface of the metal.
- Since the work function w_0 is constant for a particular metal, there exists a minimum frequency (threshold frequency, f_0) given by $w_0 = hf_0$

  From, $hf - w_0 = \frac{1}{2} mv^2$

  then $h(f-f_0) = \frac{1}{2} mv^2$

Also, $w_0 = hf_0$ and $f_0 = \frac{c}{\lambda_0}$

- $w_0 = \frac{hc}{\lambda_0}$
- If an electron of charge e is accelerated by a voltage V volts, it gains K.E given by K.E = eV.

  Hence from above $h(f - f_0) = eV$
- An electron volt (eV) is the K.E gained by an electron which has been accelerated through a p.d of one volt
- $1eV = 1.6 \times 10^{-19}J$
- The values of the constants are $h = 6.64 \times 10^{-34}Js$, $c = 3.0 \times 10^{8}ms^{-1}$, $e = 1.6 \times 10^{-19}C$

Definitions

Threshold wavelength is the maximum wavelength that is required to emit the electrons from a metal in the photo electric effect

Threshold frequency is the minimum frequency of incident radiation below which photoelectric emission cannot occur.

Example 1

Monochromatic radiation of frequency 1.0×10^{15} Hz is incident on a clean magnesium surface for which the work function is $0.59 \times 10^{-18}J$. Calculate

(i) the maximum kinetic energy of the emitted electrons

kinetic energy = $hf - w_0$

$= 1 \times 10^{15} \times 6.64 \times 10^{-34} - 0.59 \times 10^{-18}J$

$= 7.4 \times 10^{-20}J$

(ii) the potential to which the magnesium surface must be raised to prevent the escape of electrons potential energy = kinetic energy

eV = 1.04 x 10^{-19}J

V = 7.4 x 10^{-20}J/1.6 x 10^{-19}

= 0.46V

(iii) The cut-off wavelength.

From $w_0 = \frac{hc}{\lambda_0}$

$$\lambda_0 = \frac{6.64\ x\ 10^{-34}\ x\ 3\ x\ 10^8}{0.59\ x\ 10^{-18}}\ 3.38 \text{ x } 10^{-7}\text{m}$$

Example 2

Calcium has a work function of 2.7eV.

a) What is the work function of calcium in Joules?

1eV = 1.6 x 10^{-19}J

∴ 2.7eV = 2.7 x 1.6 x 10^{-19} = 4.3 x 10^{-19}J

b) What is the threshold frequency of calcium?

hf_0 = 4.3 x 10^{-19}

6.64 x 10^{-34} x f_0 = 4.3 x 10^{-19}

f_0 = 6.5 x 10^{14}Hz

c) What is the maximum wavelength that will cause emission from calcium metal?

$$\lambda_0 = \frac{C}{f_0} = \frac{3\ x\ 10^8}{6.5\ x\ 10^{14}} = 4.6 \text{ x } 10^{-7}\text{m}$$

Example 3

Light of frequency 6 x 10^{14}Hz is incident on a metal surface and the emitted electrons have kinetic energy of 2 x 10^{-29}J. Calculate:

(i) Work function

From hf = $w_0 + \frac{1}{2}mv^2$

6 .63 x 10^{-34} x 6 x 10^{14} = w_0 +2 x 10^{-29}

w_0 = 3.978 x 10^{-19}J

(ii) Threshold frequency of the metal.

From $w_0 = hf_0$

3.978 x 10^{-19} = 6 .63 x 10^{-34} x f_0

f_0 = 6 x 10^{14}Hz

Example 4

Calculate the maximum speed of photoelectrons emitted by a cesium surface when irradiated with light of wavelength 484mm if the work function of cesium is 3 x 10^{-19}J.

(c = 3 x 10^8ms^{-1}, h= 6.63 x 10^{-34}Js, Me = 9.1 x 10^{-31}kg)

From hf = $w_0 + \frac{1}{2}mv^2$

$\frac{hc}{\lambda} = w_0 + \frac{1}{2}mv^2$

$$\frac{6.63\ x\ 10^{-34}\ x\ 3\ x\ 10^8}{484\ x\ 10^{-3}} = 3\ x\ 10^{-19} + \frac{1}{2}\ x\ 9.1\ x\ 10^{-31} x\ v^2$$

v = 4.938 x 10^5ms^{-1}

Example 5

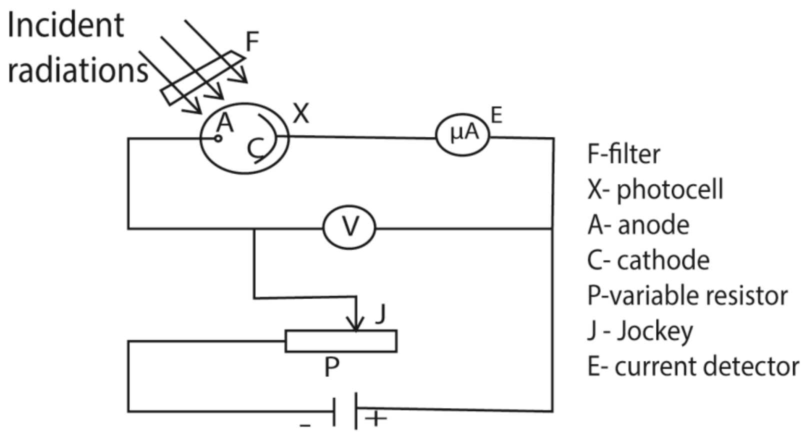
A photo emissive metal has a threshold wavelength of 0.45µm. Calculate the kinetic energy of emitted electrons when light of wavelength 0.35µm falls on this metal

($c = 3 \times 10^8 ms^{-1}$, $h = 6.63 \times 10^{-34} Js$)

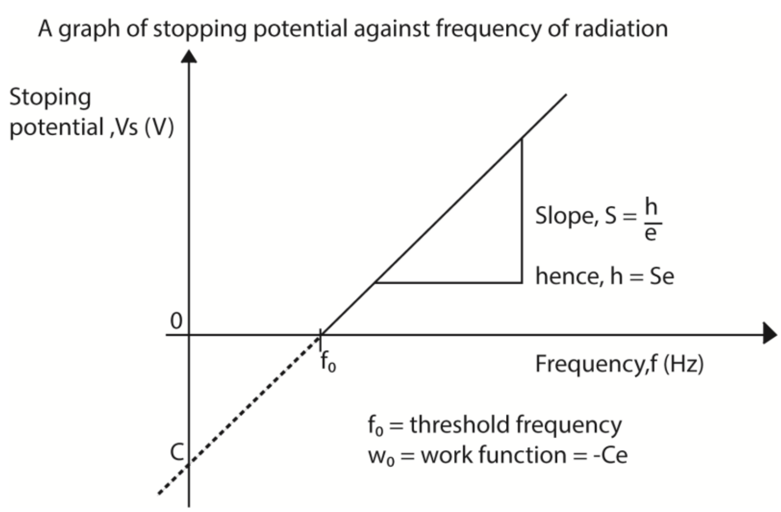
From $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + K.E$

$K.E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.35 \times 10^{-6}} - \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-6}} = 1.263 \times 10^{-19} J$

Experiment to verify Einstein's photoelectric equation and determination of Planks constant h



- A radiation of known frequency, f, is made incident on the photocathode
- Emitted electrons travel to the anode and cause a current to flow, detected at E.
- The p.d V is adjusted until the reading of E is zero (i.e. no current flows).
- The value of this p.d is the stopping potential (Vs) and is recorded from the voltmeter V.
- The procedure is repeated with light of different frequencies, f.
- A graph of stopping potential (Vs) against frequency (f) is plotted



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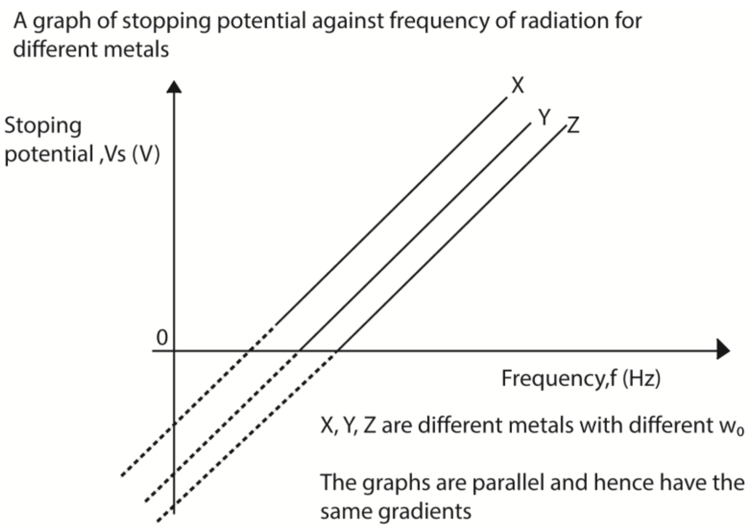
The nature of the curve verifies the equation. $V_s = \frac{h}{e}f - \frac{h}{e}f_0$

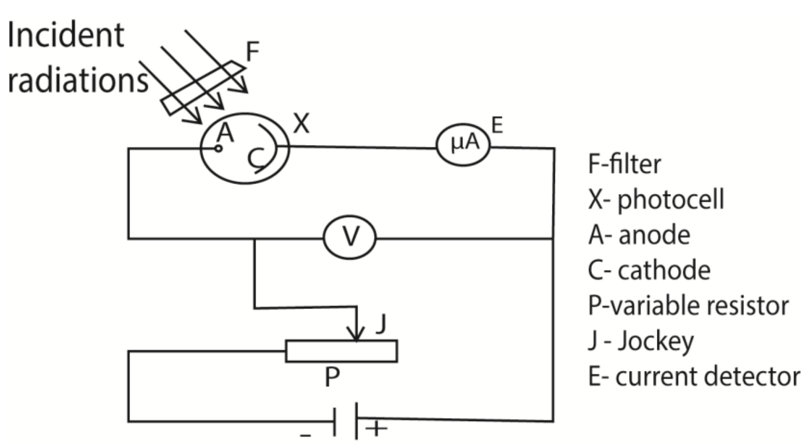
Also $\lambda_0 = \frac{C}{f_0}$

NOTE: For all different types of metals, the slope of the graph of Vs against frequency f is constant (the same i.e. $\frac{h}{e}$)



Stopping potential is the minimum potential between the cathode and the anode that prevents the most energetic electrons from reaching the anode.

Experiment to measure stopping potential of a metal



- An evacuated electric cell X that has inside it a photo-emissive metal cathode, C of large surface area and an anode A for collecting the electron produced
- A is made negative in potential relative to C.
- The photoelectrons emitted from C when illuminated with a suitable beam experience a retarding potential.
- The p.d V is increased negatively until the current become zero and the stopping potential Vs is noted from the voltmeter.

Example 6

Sodium has a work function of 2.3eV. Calculate the

(i) Threshold frequency

From $w_0 = hf_0$

$2.3 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} f_0$

$f_0 = 5.55 \times 10^{14}$Hz

(ii) Stopping potential when it is illuminated by light of wavelength 5×10^{-7}m ($1eV = 1.6 \times 10^{-19}J$)

From $hf = hf_0 + eV$

$$V = \frac{h(f - f_0)}{e} = \frac{6.63 \times 10^{-34}\left(\frac{3 \times 10^{8}}{5 \times 10^{-7}} - 5.55 \times 10^{14}\right)}{1.6 \times 10^{-19}} = 0.186V$$

Explanation of the laws of photoelectric emission using quantum theory

The quantum theory states that "light is emitted and absorbed in discrete packets of energy called photons"

When light is incident on a metal surface, each photon interacts with a single electrons giving it all its energy. The photon is absorbed if its energy is greater than the work function and if it is less, the photon is rejected.

Increasing the intensity of light increases the number of photons striking the metal surface per second. Therefore more electrons are emitted per second and the photocurrent increases with intensity.

Increasing the frequency of incident radiation increases the energy of each photon and therefore the maximum kinetic energy of the liberated electrons increases with the frequency of radiation.

Increasing the intensity of light only increases the number of photons but not the energy in each photon. Hence kinetic energy of the emitted electrons is independent of the intensity of the incident radiation

Failures of the wave theory (classical theory) to account for the photoelectric emission

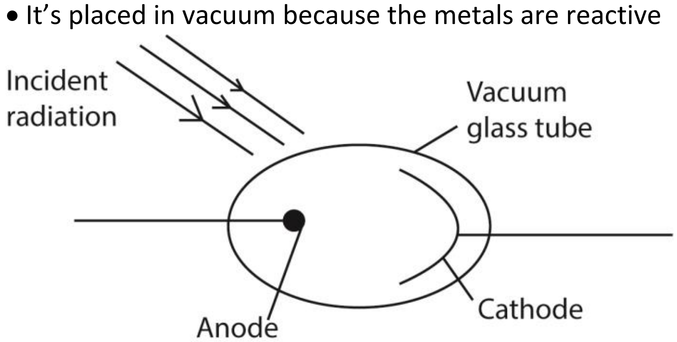
1. **Existence of threshold frequency**
   According to the classical theory, the energy of the incident radiation depends on its intensity; the greater the intensity of illumination, the greater the supply of energy. This would imply that radiations of high enough intensity should cause emission even when the frequency is below the minimum value. However as long as the incident radiation is below the threshold frequency, no photoelectrons are emitted however intense the incident radiation is
2. **Instantaneous emission of photoelectrons**
   Classical theory suggests that the energy of the incident radiation would be continuously absorbed by the electron. Implying that the electron would take some time to accumulate

sufficient energy that would enable them escape from the metal surface. By this theory, emission of photoelectrons would not be instant

3. **Variation of K.E of the emitted photoelectrons**
   According to the classical theory, increasing the intensity of the incident radiation would mean more incident energy and a greater maximum K.E of the emitted photoelectrons. But instead the maximum K.E of the photoelectrons emitted depend on the frequency of the incident radiation.
4. **Variation of photoelectric current with intensity**
   When the intensity of illumination is increased, the number of photons incident on the metal surface also increases. Hence more free electrons in the metal receive sufficient energy to escape. The rate of emission increases and therefore a large current flows. Thus the size of the photocurrent depends on the intensity of the incident radiation. However, According to classical theory, increase in the intensity would increase the K.E of the emitted electron and they would escape with greater speed instead, which is false

The Photocell

- Photocells change radiation into electric current.
- In their construction, the anode is made thin so that it does not obstruct the incident radiation
- It's placed in vacuum because the metals are reactive

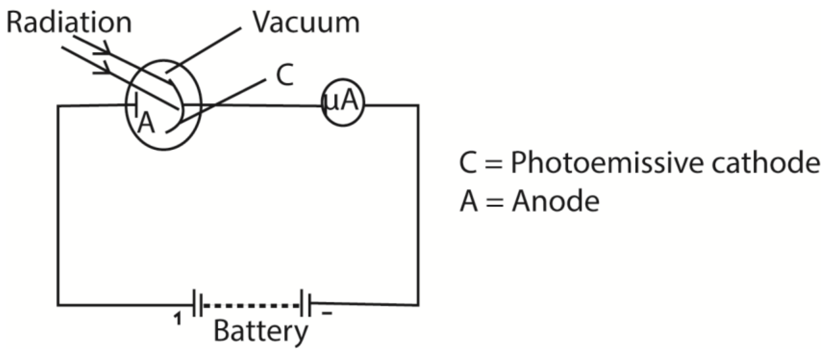


- When radiations fall on the cathode, electrons are emitted and these electrons are collected by the anode.
- If the anode is positive respect to the cathode, current flows in the circuit

Types of Photocells

(a) Photo emissive cells
(b) Photovoltaic cells
(c) Photoconductive cells.

Photo emissive Cell



- When radiation of frequency f greater than f_0 (threshold frequency) of the photo emissive cathode is incident on the cathode, electrons are emitted, they move to the anode and current flows in the external circuit.
- The size of the current increases with the intensity of the incident radiation.
- If the light beam is interrupted, the current stops flowing.
- When the device is connected to a suitable relay circuit, it can be used to open doors, act as a burglar alarm or as switching device.

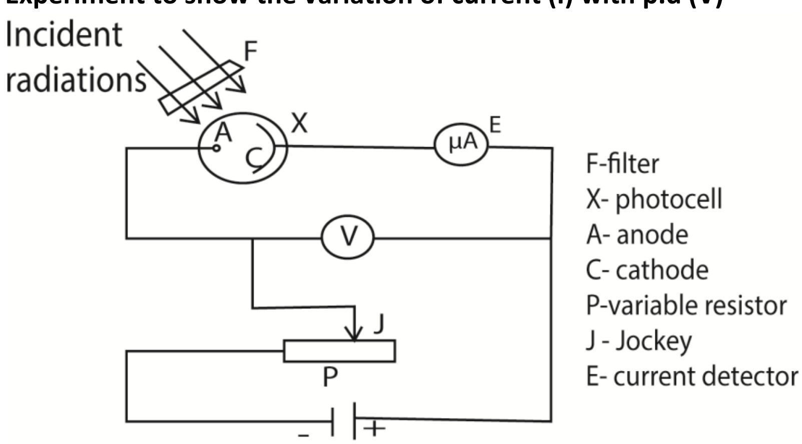
Photovoltaic Cell

It generates an e.m.f dependent on the intensity of the incident radiation. Such cells are used in solar panels, solar calculators and for powering electronic watches.

Photoconductive Cell

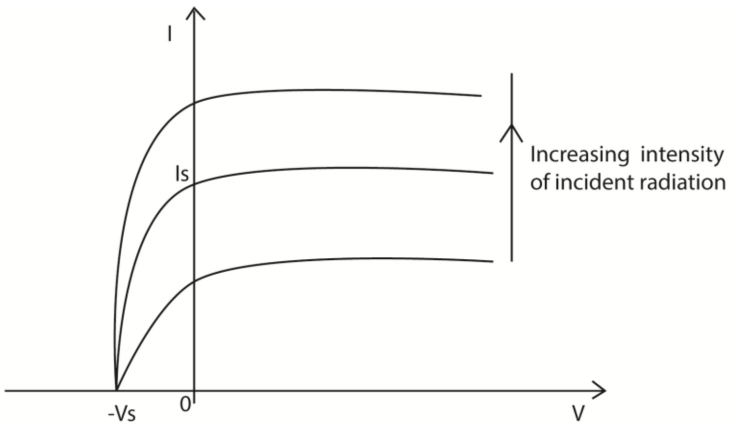
It consists of a plate of a material called photoconductor, whose resistance decreases when it is illuminated by light or infrared radiation, mounted in an evacuated glass bulb. An applied voltage causes current to flow which increases with the intensity of the radiation due to release of electrons in the photoconductor.

Experiment to show the variation of current (I) with p.d (V)



- A monochromatic light i.e. constant frequency is used.
- The photocurrent (I) is measured for increasing values of V at constant light intensity.
- For negative values of V, the polarity of the battery is reversed.

- The experiment is repeated by increasing the intensity of the radiations; by moving the light source closer to the photocell
- A plot of graphs of photocurrent (I) against the p.d V is shown below.

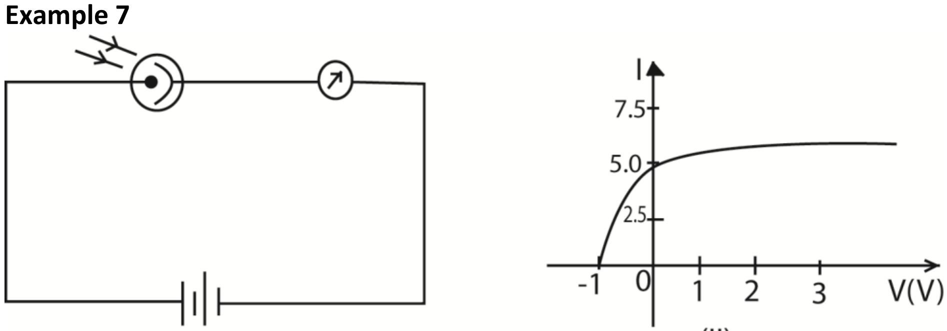


Is = saturation current at that intensity
Vs = the stopping potential for the cathode

-

N.B: The photocurrent is not zero even when the p.d is zero. This is because electrons are emitted with varying velocities (K.E), some of which are sufficient to overcome the repulsive electric field and reach the anode.

Example 7



(I) (II)

A photocell is connected in the circuit as shown in figure (I) above. The cathode is illuminated with monochromatic light of wavelength 390nm and the current I in the circuit recorded for different p.d V applied between the anode and the cathode. The graph fig (II) shows the results obtained.

(a) Find the maximum K.E of the photoelectrons

From the graph Vs = -1V

$K.E_{max} = eV = 1.0 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19}$

(b) What is the work function of the cathode in eV?

$K.E_{max} = \frac{hc}{\lambda} - w_0$

$w_0 = \frac{hc}{e\lambda} - K.E_{max}$

$= \frac{6.64 \times 10^{-34} \times 3.0 \times 10^{8}}{1.6 \times 10^{-19} \times 390 \times 10^{-9}} - 1$

= 2.19eV

(c) If the experiment is repeated using monochromatic light of wavelength 310nm, where would the new graph cut the V-axis?

$K.E_{max} = \frac{hc}{e\lambda} - w_0$

$= \frac{6.64 \times 10^{-34} \times 3.0 \times 10^{8}}{1.6 \times 10^{-19} \times 310 \times 10^{-9}} - 2.19$

= 1.83eV

Hence the graph would cut the v-axis at – 1.83V

Applications of Photocells

(i) Photoelectric Emission)

(ii) A photocell can make doors open automatically in buildings when a light beam is interrupted by somebody/obstacle.

(iii) Intruder alarm systems. The intruder intercepts the infrared beam falling on a photocell, hence cutting off of current. This interruption therefore sets the alarm on.

(iv) Photovoltaic cells are used in solar panels, calculators and for powering electronic watches.

(v) Used as automatic devices for switching on light at night when it tries to darken or when the frequency of the light reduces.

(vi) Automatic counting machines in industries.

(vii) Production of sound from a film

Differences between x-ray production and photoelectric effect:

Photoelectric effect	X-rays
Electromagnetic radiation falls on metal surface and electrons are emitted	Fast moving electrons hit the metal target and x-rays (electromagnetic radiation) is produced
Little heat is generated	A lot of heat is generated

Example 8

A 100mW beam of light of wave length 4.0×10^{-7}m falls on a caesium surface of a photocell.

(i) How many photons strike the caesium surface per second?

$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{4.0 \times 10^{-7}} = 4.98 \times 10^{-19}$J

Number of photons, $n = \frac{total\ energy}{Energy\ of\ one\ photon}$

$= \frac{100 \times 10^{-3}}{4.98 \times 10^{-19}}$

$= 2 \times 10^{17} s^{-1}$

(ii) If 80% of the photons emit photoelectrons. Find the resulting photocurrent.

Number of electrons emitted = $2 \times 10^{17} s^{-1} \times 80\%$

$= 1.6 \times 10^{17}$

Current = ne

$= 1.6 \times 10^{17} \times 1.6 \times 10^{-19}$

$= 2.56 \times 10^{-2}$A

(iii) Calculate the kinetic energy of each photoelectron if the work function of caesium is 2.15eV.

$$K.E_{max} = \frac{hc}{e\lambda} - w_0$$
$$= \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{4.0 \times 10^{-7} \times 1.6 \times 10^{-19}} - 2.15$$
$$= 0.96 eV$$

Experimental evidence for quantum theory

(i) **Photoelectric effect:**

To liberate an electron from a metal surface, a quantum or packet of energy called the work function which is characteristic of the metal surface has to be supplied

i.e. $hf - w_0 = \frac{1}{2}mv^2$ where w_0 is the work function.

(ii) Optical spectra:

A line in the optical emission spectrum indicates the presence of a particular frequency f of light and is considered to arise from loss of energy which occurs in an excited atom when an electron jumps directly or in steps from a higher energy level E_2 to lower energy level E_1. The frequency of the packet of energy emitted is given by $hf = E_2 - E_1$.

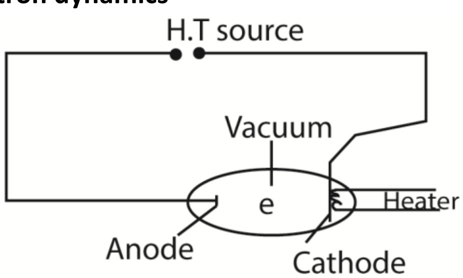
(iii) X-ray line spectra:

Electron transition from one shell to another leads to liberation of energy in packets characteristic of the target atom.

Differences between classical theory and quantum theory

Classical(wave) theory	Quantum theory
It allows continuous absorption and accumulation of energy.	No continuous absorption allowed. The energy is either absorbed or rejected.
Energy of radiation is evenly distributed over the wave front.	Energy is radiated, propagated and absorbed in packets (quanta or photons).
What matters is total energy of the incident radiation (beam).	What matters is the energy of individual photon.

Electron dynamics



Consider an electron moving from cathode to Anode.

- Let the p.d between the cathode and anode be V. The electron will be accelerated by the electric field and hence it gains K.E.
- If the electron starts from the cathode with zero velocity and reaches the anode with velocity ums^{-1}, then the K.E gained by

$\frac{1}{2}mu^2 = eV$ where e = electron charge, $1.6 \times 10^{-19}C$

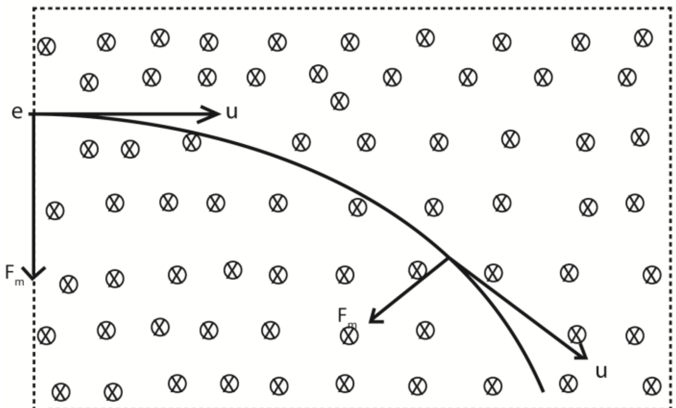
Or

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{2\left(\frac{e}{m}\right)V}$$

The quantity $\left(\frac{e}{m}\right)$ is called the specific charge of the electron

Deflection of an electron in a magnetic field

Consider an electron entering a uniform magnetic field of flux B, at right angles to its direction of motion with velocity u.



When the electron enters the field, the magnitude of its speed u does not change because the magnetic force is perpendicular to the direction of the electron, But instead its direction changes and the electron moves in a circular arc.

Let r be the radius of the circular arc (path)

The Centripetal force on the electron, F = $\frac{mu^2}{r}$ ……………………………….. (i)

The force due to the magnetic field, F = Beu ……………………………….. (ii)

From (i) and (ii)

$$\frac{mu^2}{r} = Beu$$

$$r = \frac{mu}{Be}$$

Since the speed of the electron is constant, its K.E is also constant and expressed as kinetic energy

Kinetic energy = $\frac{1}{2}mv^2 = \frac{e^2B^2r^2}{2m}$

Example 9

An electron moves in a circular path at 3.0 x 10^6m/s in a uniform magnetic field of flux 2.0 x 10^{-4}T. Find the radius of the path.

[mass of an electron m_e = 9.11 x 10^{-31}kg, e = 1.6 x 10^{-19}C]

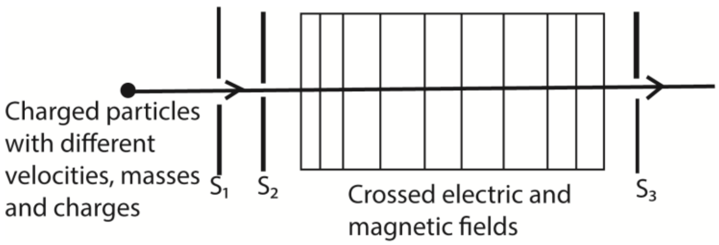
$$r = \frac{mu}{Be}$$

$$= \frac{9.11 \times 10^{-31} \times 3.0 \times 10^{6}}{2.0 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

= 8.5cm

Crossed fields

Crossed fields are fields in which a uniform magnetic field and a uniform electric field are perpendicular to each other producing deflections opposite to each other. If the magnetic force and electric force in the crossed fields are of the same magnitude, there is no deflection on charged particles that enter such fields.



The slits S_1 and S_2 confine the particles into a narrow beam as they enter the crossed fields. The only particles that emerge at slit S_3 are those which are undeflected, and therefore they emerge with the same velocity u.

The electric force F_E due to the electric field = eE

The magnetic force F_m due to the magnetic field = Beu

For crossed fields $F_E = F_m$

$$eE = Beu$$

$\therefore u = \frac{E}{B}$ this is the velocity of particles emerging at S_3

Therefore all particles that emerge at S_3 will have the same velocity $u = \frac{E}{B}$ regardless of their mass and charge.

The crossed fields can be used as a velocity selector of particles of a single velocity from a beam of particles of different velocities.

Example 10

An electron accelerated by a p.d of 1.5KV passes through an electric field crossed with a uniform magnetic field of flux density 0.45T. Calculate the value of the electric field needed for the electron to emerge undeflected.

Solution

$u = \frac{E}{B}$

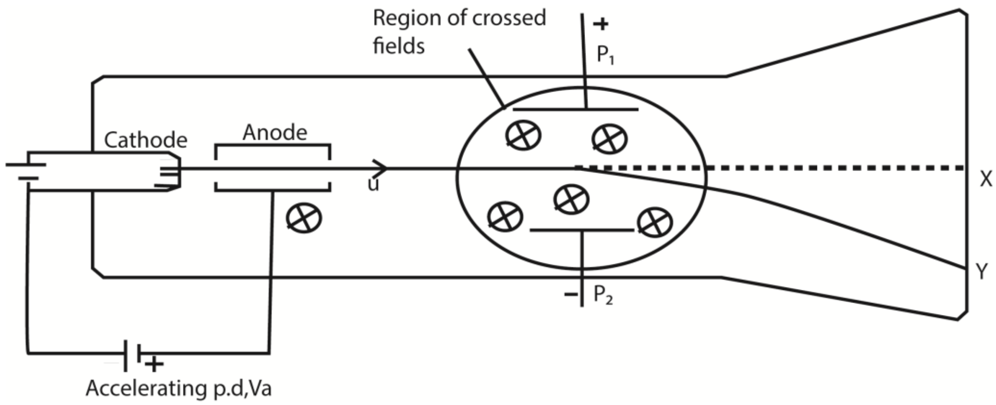
Also, $\frac{1}{2}mu^2 = eV$

$$u = \sqrt{\frac{2eV}{m}}$$

$$\therefore \frac{E}{B} = \sqrt{\frac{2eV}{m}}$$

$$E = B\sqrt{\frac{2eV}{m}} = 0.45\sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1.4 \times 10^{3}}{9.11 \times 10^{-31}}} = 1.033 \times 10^{7}\,NC^{-1}$$

Determination of Specific Charge $\left(\frac{e}{m}\right)$ of an electron: (J.J Thomson's Method)



- The electrons are produced thermionically by a hot filament cathode and are accelerated towards a cylindrical anode and pass through it.
- The small hole on the anode confines the electrons to a narrow beam.
- When both the electric field and the magnetic field are off, the electrons reach the screen at X and cause fluorescence.
- If the velocity of the electrons on emerging from the anode is u then

  eVa = $\frac{1}{2}mu^2$

  ⇨ $\frac{e}{m} = \frac{u^2}{2Va}$ .... (i)

  Where Va is the accelerating voltage between the cathode and anode.

- The magnetic field is switched on and the beam is deflected to position Y.
- In order to bring the beam back to the original position X, the electric field is switched on and adjusted until the beam is at X again.
- This implies that The magnetic force = the electric force

  Beu = eE

  ∴u = $\frac{E}{B}$ .... (ii)

  Substituting eqn. (ii) in (i)

  $\frac{e}{m} = \frac{E^2}{2B^2Va}$ but E = $\frac{V}{d}$

  ∴ $\frac{e}{m} = \frac{V^2}{2B^2d^2Va}$ where, V is the p.d between the plates at separation of d apart

Example 11

A beam of electrons is accelerated through a p.d of 500V and enters a uniform electric field of strength 3.0×10^3V/m created by two parallel plates of length 2.0cm. Calculate:

(a). the speed of the electrons as they enter the field.

From $\frac{1}{2}mu^2 = eV$

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \ x \ 1.6 \ x \ 10^{-19} \ x \ 500}{9.11 \ x \ 10^{-31}}} = 1.325 \times 10^7 ms^{-1}$$

(b). the time that each electron spends in the field.

$$t = \frac{L}{u} = \frac{2.0 \ x \ 10^{-2}}{1.325 \ x \ 10^7} = 1.51 \times 10^{-9}s$$

(c). the angle from which the electrons have been deflected by the time they emerge from the field.

$u_x = u = 1.325 \times 10^7 ms^{-1}$

$u_y = a_y t$ but $ay = \frac{eE}{m}$

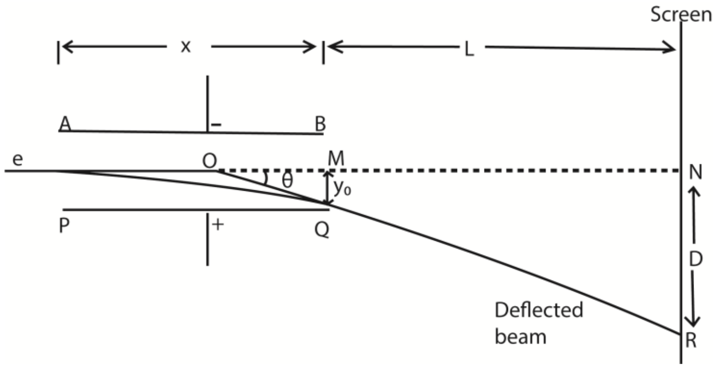
$\therefore u_y = \frac{eEt}{m} = \frac{1.6 \times 10^{-19} \times 3 \times 10^3 \times 1.51 \times 10^{-9}}{9.11 \times 10^{-31}} = 7.956 \times 10^5 ms^{-1}$

Let the angle be θ

$\tan \theta = \frac{u_y}{u_x} = \frac{7.956 \times 10^5}{1.3225 \times 10^7} = 3.4^0$

Motion of an electron in an electric field

Consider two parallel plates AB and PQ such that AB is vertically above PQ and at a distance d apart. Let l be the length of the plates and V the p.d between the plates.



This electric force is directed towards the positive plate causing the deflection of the beam as shown above.

But for parallel plates $E = \frac{V}{d}$

Thus $F_E = \frac{eV}{d}$

Since the electric field intensity is vertical, there is no horizontal force acting on the electron. Hence the horizontal component of the velocity of the electron does not change.

Let u be the horizontal component of the velocity of the electron entering the electric field.

Motion in the X-direction

$s = ut + ½ at^2$, but $s = x$ and $a = 0$

⇨ $t = \frac{x}{u}$……………………………… (i)

Motion in y-direction

$s = ut + ½ at^2$, but $uy = 0$ $s = y$ and $a = \frac{eE}{m}$ from $ma = eE$

$y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{x}{u}\right)^2$

$= \left(\frac{eE}{2mu^2}\right) x^2$

⇨ $y \propto x^2$ or $y = kx^2$ which is an equation for parabola

Thus the motion of an electron in electric field is parabolic

Note

1. The time, t, taken for the electron to pass through the electric field (leave the plates)
   t = $\frac{x}{u}$
2. The velocity, V_0 with which electrons leave the plates

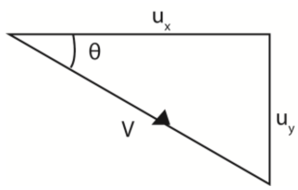
   $V_0 = \sqrt{v_x^2 + v_y^2}$

   $u_x = u \ and$

$from\ v_y = u_y + a_y t, but\ u_y = 0\ and\ a_y = \frac{eE}{m}$ also t = $\frac{x}{u}$ where x is the length of the plates

Thus $v_y = \left(\frac{eE}{m}\right)\frac{x}{u}$ where E = $\frac{V}{d}$

3. Direction (angle) the electron emerges from thee region between the plates at angle θ given by



   $\tan\theta = \frac{v_y}{v_x}$
4. The deflection D of the electron on the screen placed at a distance L from the edge of the plates is obtained from
   tan θ = $\frac{D}{L+\frac{1}{2}X} = \frac{eEx}{mu^2}$
   hence D = $\frac{eEx}{mu^2}(L + \frac{1}{2}x)$ where E = $\frac{V}{d}$

Example 12

A beam of electrons is accelerated through a p.cd of 2000V and is directed mid-way between two horizontal parallel plates of length 5.0cm and separation of 2.0cm. The p.d across the plates is 80V.

(a). Calculate the speed of the electrons as they enter the region of between the plates

From $\frac{1}{2}mu^2$ = eV

u = $\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\ x\ 1.6\ x\ 10^{-19}\ x\ 2000}{9.11\ x\ 10^{-31}}}$ = 2.65 x $10^7$$ms^{-1}$

(b). Find the speed of the electrons as they emerge from the region between the plates.

$u_y = a_y t\ but\ a_y = \frac{eE}{m}, E = \frac{V}{d}\ and\ t = \frac{x}{u_x}$

$v_y = \left(\frac{eV}{md}\right)\left(\frac{x}{u_x}\right) = \frac{1.6\ x\ 10^{-19}\ x\ 80\ x\ 5\ x\ 10^{-2}}{9.11\ x\ 10^{-31}\ x\ 2\ x\ 10^{-2}\ x\ 2.65\ x10^7}$ = 1.325 x $10^6$$ms^{-1}$

$V = \sqrt{u_x^2 + u_y^2}$ = 2.653 x 107ms^{-1}

(c). Explain the motion of the electrons between the plates.

It is parabolic

Example 9

An electron gun operated at $3x10^3$V is used to project electrons into the space between two oppositely charged parallel plates of length 10cm and separation 5cm. Calculate the deflection of the electrons as they emerge from the region between the charged plates when the p.d is $1.0x10^3$V.

From ½ $mu^2 = eV$

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\ x\ 1.6\ x\ 10^{-19}\ x\ 3000}{9.11\ x\ 10^{-31}}} = 3.246 \text{ x } 10^7 ms^{-1}$$

Also $y = u_y t + \frac{1}{2}a_y t^2$ but $u_y = 0, a_y = \frac{eE}{m}; t = \frac{x}{u}$

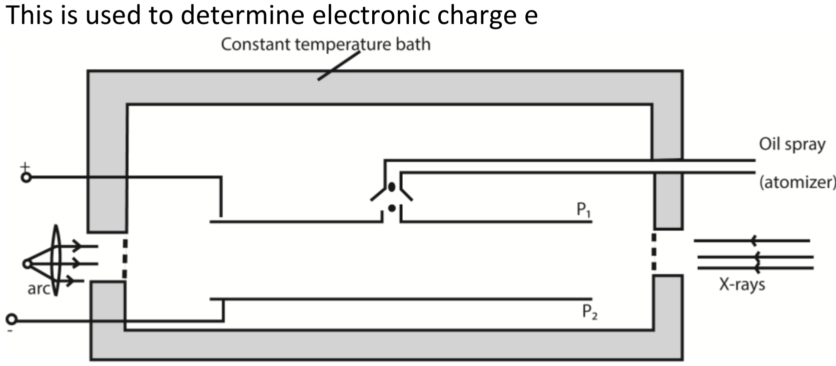
⇨ $y = \frac{1}{2}\frac{eEx^2}{mu_x^2}$ and $E = \frac{V}{d}$

⇨ $y = \frac{1}{2}\frac{eVx^2}{mdu_x^2} = \frac{1.6\ x\ 10^{-19}\ x\ 1000\ x\ (10\ x\ 10^{-2})^2}{2\ x\ 9.11\ x\ 10^{-31}\ x\ 5\ x\ 10^{-2}\ x\ (3.246\ x\ 10^7)^2} = 1.667 \text{ x } 10^{-2}m$

Therefore, the deflection= $1.667 \text{ x } 10^{-2}$m

Millikan's Oil drop experiment

This is used to determine electronic charge e

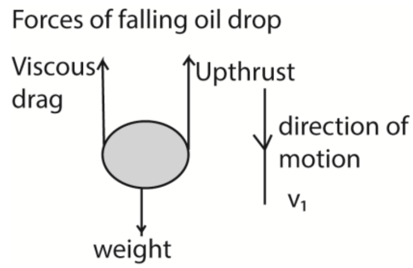


Procedure

- Set up of the apparatus is as shown above
- Oil drops are introduced between the plates P_1 and P_2 by spraying using the atomizer.
- These oil drops are charged in the process of spraying by friction but the charge may be increased further ionization due to X-rays.
- The oil drops are strongly illuminated by an intense light from the arc lamp so that they appear as bright spots when observed through a low power microscope.
- With no electric field between the plates, record the time t_1 taken for drop to fall from P_1 to P_2.
- The electric field between the plates is turned on and adjusted so that the drop becomes stationary.

Case 1

With no electric field, the oil drop falls with a uniform velocity v_1 called terminal velocity



Weight = Upthrust + viscous drag ……………………………… (i)

= volume of the oil drop x density x gravity

= $\frac{4}{3}\pi r^3 \rho g$ (ρ = density of oil, r = radius of oil drop)

Upthrust = weight of air displaced by oil drop

= volume of the air displaced by oil drop x density x gravity

= $\frac{4}{3}\pi r^3 \sigma g$ (σ = density of air)

Viscous drag = $6\pi r\eta v_1$ (From strokes' law)

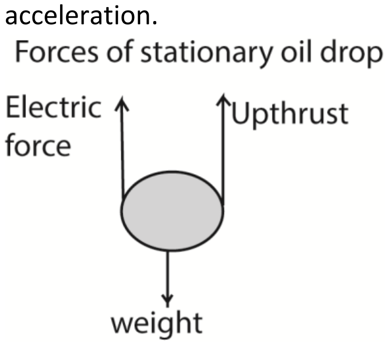
From 1

$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r\eta v_1$ …………………… (ii)

$r = \left[\frac{9\eta v_1}{2g(\rho-\sigma)}\right]^{\frac{1}{2}}$

Case 2

When the electric field is applied so that the drop is stationary, the drop has no velocity and no acceleration.



Weight = Upthrust + electric force

$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + qE$ …………………… (iii)

From (ii) and (iii)

q = $\frac{6\pi r\eta v_1}{E}$ but E = $\frac{V}{d}$

Substituting for r

q = $\frac{6\pi\eta d v_1}{V}\left[\frac{9\eta v_1}{2g(\rho-\sigma)}\right]^{\frac{1}{2}}$

Note: the density of air at room temperature is very small compared to that of oil and thus maybe assumed negligible (when it's not given) in calculations. This implies that the up thrust due to the displaced air is Zero.

q = $\frac{6\pi\eta d v_1}{V}\left[\frac{9\eta v_1}{2g\rho}\right]^{\frac{1}{2}}$

Precautions

To improve on the accuracy of the experiment, the following precautions need to be taken into account

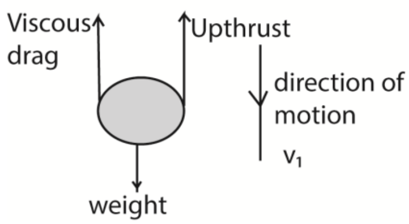
1) A non-volatile oil or low vapor pressure oil should be used to reduce evaporation. Evaporation would alter the mass of the drop
2) The experiment is enclosed in a constant temperature enclosure. This is to eliminate convection currents and changes in the viscosity of air as a result of temperature changes.
3) An X-ray tube is used to increase the charge of the oil drop.

Example 14

Oil droplets are introduced into the space between two flat horizontal plates, set 5.0mm apart. The plate voltage is then adjusted to exactly 780V so that one of the droplets is held stationary. Then the plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.5mm in 11.2s. Given that the density of the oil used is 900kgm^{-3} and the viscosity of air is 1.8 X 10^{-5}Nsm^{-2}, find;

i) The radius of the oil drop

Forces of falling oil drop



Weight = Upthrust + viscous drag

$$\frac{4}{3}\pi r^3\rho g = \frac{4}{3}\pi r^3\sigma g + 6\pi r\eta v_1$$

Density of air is negligible, σ = 0

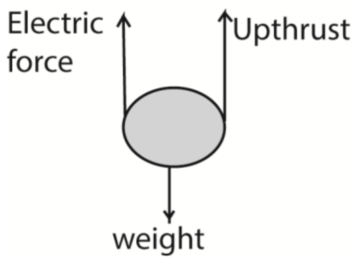
$$r = \left[\frac{9\eta v_1}{2g\rho}\right]^{\frac{1}{2}}$$

Velocity, $v_1 = \frac{distance\ travelled\ by\ oil\ drop}{time} = \frac{1.5\ x\ 10^{-3}}{11.2}$ 1.339 x 10^{-4}ms^{-1}

$$\therefore r = \left[\frac{9\ x\ 1.8\ x\ 10^{-5}\ x\ 1.339\ x\ 10^{-4}}{2\ x\ 9.81\ x\ 900}\right]^{\frac{1}{2}} = 1.1 \text{ x } 10^{-6}\text{m}$$

ii) The charge of the droplets

Forces of stationary oil drop



Weight = Upthrust + electric force

$\frac{4}{3}\pi r^3\rho g = \frac{4}{3}\pi r^3\sigma g + \text{qE}$

But σ = 0 (can be neglected, E = $\frac{V}{d}$

q = $\frac{4\pi r^3\rho g d}{3v}$ = $\frac{4\pi\left(1.1 \; x \; 10^{-6}\right)^3 \; x \; 900 \; x \; 9.81 \; x \; 5 \; x \; 10^{-3}}{3 \; x \; 780}$ = 3.2 x 10^{-19}C

iii) The number of electrons on the drop

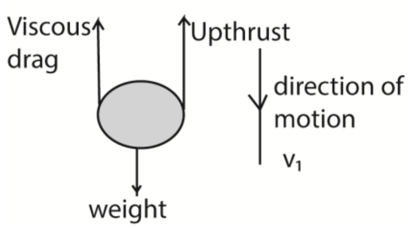
$n = \frac{q}{e} = \frac{3.2 \; x \; 10^{-19}}{1.6 \; x \; 10^{-19}}$ = 2

thus the drop has acquired 2 electrons

Example 15

a) Calculate the radius of a drop of oil of density 900kgm^{-3} which falls with a terminal velocity of 2.9 x 10^{-4}m/s through air of viscosity 1.8 X 10^{-5}Nsm^{-2}.

Forces of falling oil drop



Weight = Upthrust + viscous drag

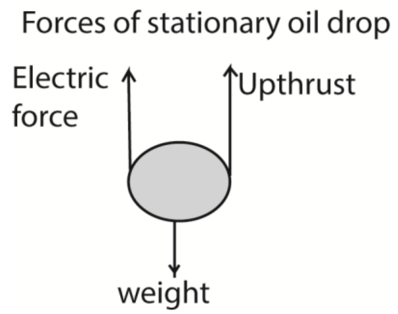
$\frac{4}{3}\pi r^3\rho g = \frac{4}{3}\pi r^3\sigma g + 6\pi r\eta v_1$

Density of air is negligible, σ = 0

$r = \left[\frac{9\eta v_1}{2g\rho}\right]^{\frac{1}{2}}$

$r = \left[\frac{9 \; x \; 1.8 \; x \; 10^{-5} \; x \; 2.9 \; x \; 10^{-4}}{2 \; x \; 9.81 \; x \; 900}\right]^{\frac{1}{2}}$ = 1.63 x 10^{-6}m

b) If the charge on the drop is -3e, what p.d must be applied between the plates 5.0mm apart in order to keep the drop stationary?



$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + \mathrm{qE}$

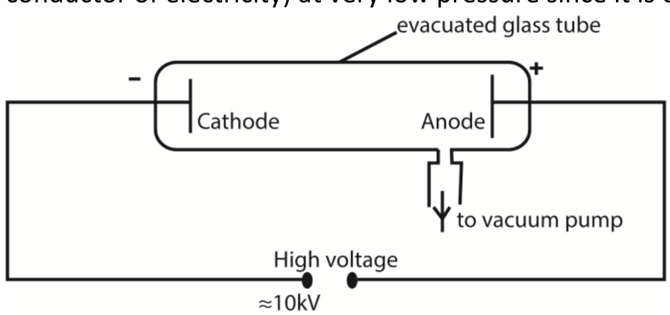
But σ = 0 (can be neglected, E = $\frac{V}{d}$

V = $\frac{4\pi r^3 \rho g d}{3\ x\ 3e}$ = $\frac{4\pi(1.63\ x\ 10^{-6})^3\ x\ 900\ x\ 9.81\ x\ 5\ x\ 10^{-3}}{3\ x\ 3\ x\ 10^{-19}}$ = 1668V

Thermionic emission and thermionic diode

Gaseous discharge in a discharge tube

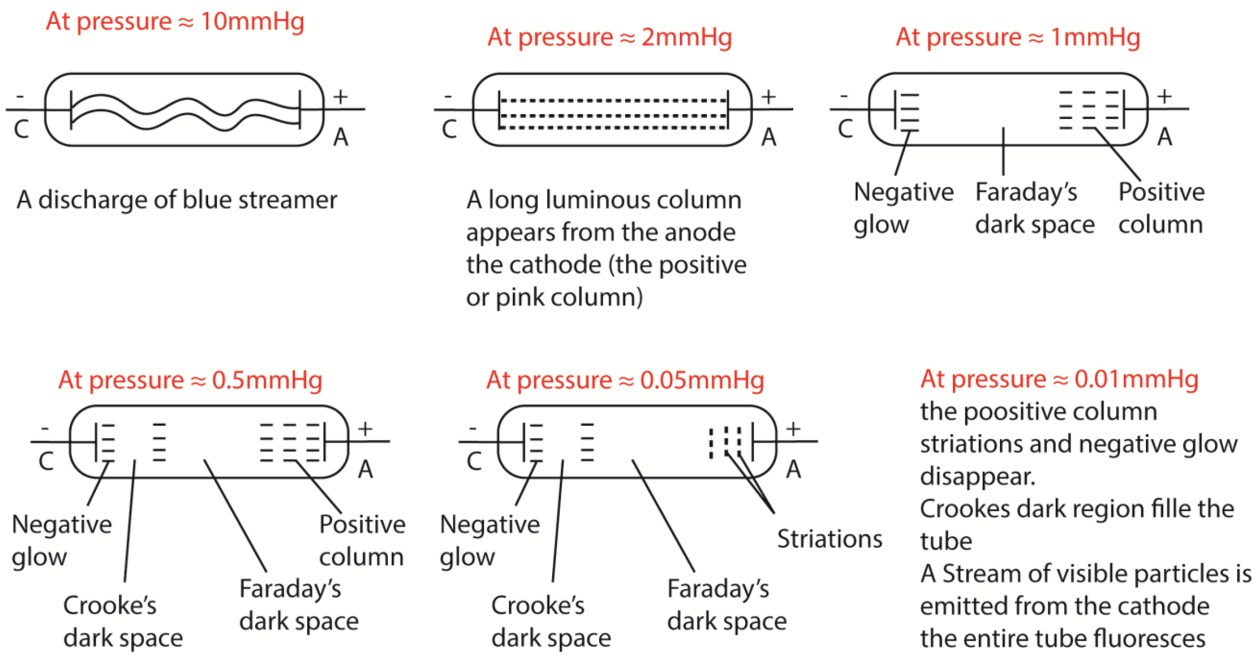
A 'discharge' is the passage of electricity through a gas at low pressure less than about 50 mm Hg.
A discharge tube is a hard glass tube connected to a vacuum pump; the tube contains two electrodes which are connected to an external high voltage source. The discharge tube contains air (which is a poor conductor of electricity) at very low pressure since it is evacuated.



Action of a discharge tube

- As the air pressure inside the tube decreases, the gas starts to ionize while the p.d inside the tube is constant.
- When one gas atom is ionized, the electron escaping from it also ionizes other gas atoms.
- Streams of positive ions and electrons are created which move to the cathode and anode respectively.
- Current is thus generated

Changes that occurs in a discharge tube as pressure is reduced to very low values.



Cathode Rays

These are streams of fast moving electrons that travel from the cathode to the anode.

Properties of cathode rays

(i) They travel in straight lines.

(ii) They are negatively charged.

(iii) They are deflected by both electric and magnetic fields.

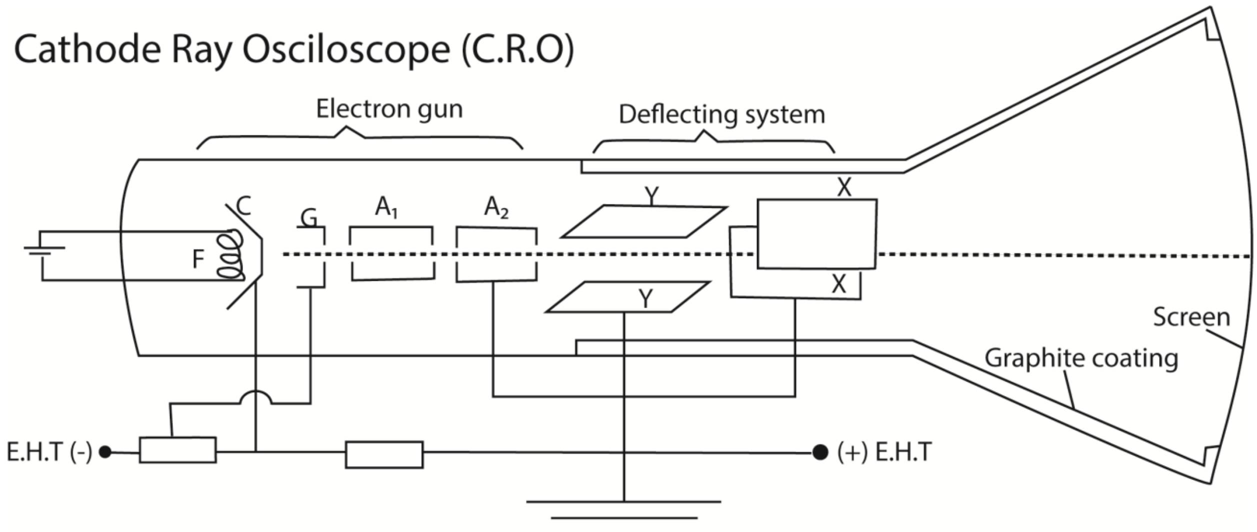
(iv) They cause fluorescence in some metals e.g. Zinc metal.

(v) They ionize gaseous atoms.

(vi) They affect photographic plates.

(vii) They produce X – rays when stopped by a metal target.

(viii) They possess momentum therefore they have mass.



Structure The C.R.O consists of three principal components or features, namely

1) The electron gun

2) The deflecting system

3) The fluorescent screen

Functions of the main parts of a C.R.O

The electron gun: this is made of the following;

- The Filament F: When a current flows through the filament, it glows and heat from the filament heats the cathode.

- The Cathode C: Emits electrons when heated by the filament.

- The Grid G: the grid is made at a negative potential with respect to the cathode. It controls the number of electrons entering the Anode, thus the Grid controls the brightness on the screen.

- Accelerating and focusing Anode A_1 and A_2: the anode is at a positive potential. It accelerates the electron beam to a high speed along the tube. Deflecting system: this consists of the X

–Plates and Y – plates;

- X – Plates are placed vertically and form a horizontal electric field. Therefore it deflects the electron beam horizontally.

- Y – Plates are placed horizontally and form a vertical electric field. Therefore it deflects the electron beam vertically.

Fluorescent screen:

This is coated with a fluorescent material such as Zinc sulphide or Phosphor. The fluorescent screen emits light (glows) when struck by fast moving electrons.

Graphite coating:

This is used:

(i) to prevent the electron beam from the influence of electric field

(ii) It collects secondary electrons emitted by the screen to the earth.

Mode of action of a C.R.O

- The cathode is heated using a low voltage supply and produces electrons by the process of thermionic emission.

- The electrons are accelerated and focused into a fine beam by the anode to fall on the screen producing a bright spot.

- The brightness of the spot on the screen is controlled by the control grid

How the grid controls the brightness of the spot of a C.R.O

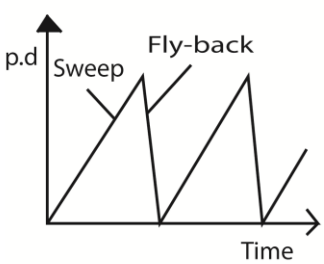
- The grid is connected to a negative potential.

- If the potential of the grid is more negative, only few electrons with high K.E (or speed) will pass through it. Thus the screen will be less bright.

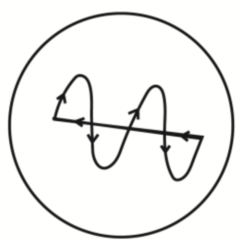
- If the potential of the grid is less negative, many electrons will pass through it. Thus the screen will be brighter.

Deflection; Time-base.

Action of a C. R. O



(i) p.d applied to X-plates



(ii) Trace of spot on screen

If a battery were connected between the Y-plates, so as to make the upper one positive, the electrons in the beam would be attracted towards that plate, and the beam would be deflected upwards. In the same way, the beam can be deflected horizontally by a potential difference applied between the X-plates.

When the oscillograph is in use, the alternating potential difference to be examined is applied between the Y-plates. If that were all, then the spot would be simply drawn out into a vertical line.

To trace the wave-form of the alternating potential difference, the X-plates are used to provide a time-axis.

A special valve circuit generates a potential difference which rises steadily to a certain value, as shown in (i), and then falls rapidly to zero; it can be made to go through these changes tens, hundreds, or thousands of times per second.

This potential difference is applied between the ·x-plates, so that the spot is swept steadily to the right, and then flies swiftly back and starts out again.

This horizontal motion provides what is called the time-base of the oscillograph.

On it is superimposed the vertical motion produced by the Y-plates; thus, as shown in Fig. ii) above, the wave-form of the potential difference to be examined is displayed on the screen.

Uses of a C.R.O

1) Measures both a.c and d.c voltages

2) Measures frequencies or compare frequencies

3) Measures phase difference

4) Measure small time intervals (used as a clock)

5) Displays wave forms

6) diagnosis heartbeat and brain in hospitals.

Comparison of CRO with a moving coil Voltmeter.

a) The C.R.O has very high impedance. It gives accurate voltages than a moving coil voltmeter.

b) A CRO can measure both d.c and a.c voltage. A moving coil voltmeter measures only D.C voltages unless a rectifier is used.

c) A CRO has negligible inertia as compared to a moving coil voltmeter. The C.R.O respond almost instantaneously.

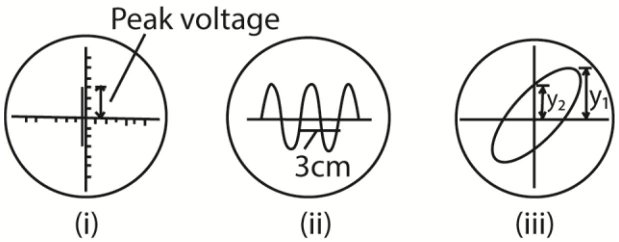
d) CRO doesn't give direct voltage readings.

Uses of Oscillograph

In addition to displaying waveforms, the oscillograph can be used for measurement of voltage, frequency and phase.

1. A.C. voltage
   An unknown a.c. voltage, whose peak value is required, is connected to the Y-plates. With the time-base switched off, the vertical line on the screen is centered and its length then measured as shown in figure (i) below. This is proportional to twice the amplitude or peak voltage, V_0. By measuring the length corresponding to a known a.c. voltage V, then V_0 can be found by proportion.

Uses of oscillograph



Alternatively, using the same gain, the waveforms of the unknown and known voltages, V and V_0, can be displayed on the screen. The ratio V/V_0 is then obtained from measurement of the respective peak-to-peak heights.

2. Comparison of frequency

   If a calibrated time-base is available, frequency measurements can be made. In Fig. (ii) above, for example, the trace shown is that of an alternating waveform with the time-base switched to the '5 millisec/cm' scale. This means that the time taken for the spot to move 1 cm horizontally across the screen is 5 milliseconds. The horizontal distance on the screen for one cycle is 3 cm. This corresponds to a time of 5 x 3 ms or 15·0 ms = 15 x 10^{-3} seconds, which is the period T.

   $\therefore frequency = \frac{1}{T} = \frac{1}{15\, x\, 10^{-3}} = 66.7Hz$

   If a comparison of frequencies, f_1, f_2 is required, then the corresponding horizontal distances on the screen are measured. Suppose these are d_1, d_2 respectively.

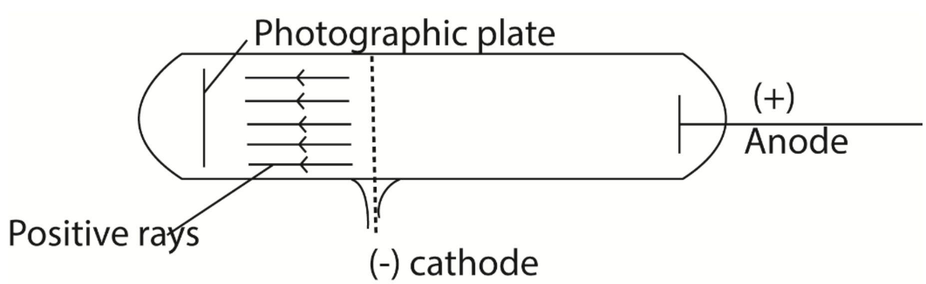
   Since $f \propto \frac{1}{T}$, then $\frac{f_1}{f_2} = \frac{T_2}{T_1} = \frac{d_2}{d_1}$

3. Measurement of phase

Positive rays

- Positive rays are streams of positively charged particles.

Production of positive rays in a discharge tube



- Positive rays are produced when a stream of electrons is passed through a vapor (gas) in discharge tube.
- The electrons dislodge electrons from the atoms producing positively charged ions.
- The positive ions are accelerated towards perforated cathode.
- The ions pass through the slits and are further accelerated.

- These ions constitute a stream of positive rays.

Properties of positive rays

1) They travel in straight lines

2) They are positively charged

3) They are more massive compared to cathode rays

4) They are reflected by strong magnetic fields

5) They have smaller specific charge (q/m) compared to the cathode rays (this is because they are more massive than the cathode rays)

6) They are deflected by strong electric fields in opposite direction to that of the cathode rays in the same field

The Bainbridge Mass Spectrometer

Consists of three main parts, namely

- Accelerating electric field

- Velocity selector

- Deflecting chamber

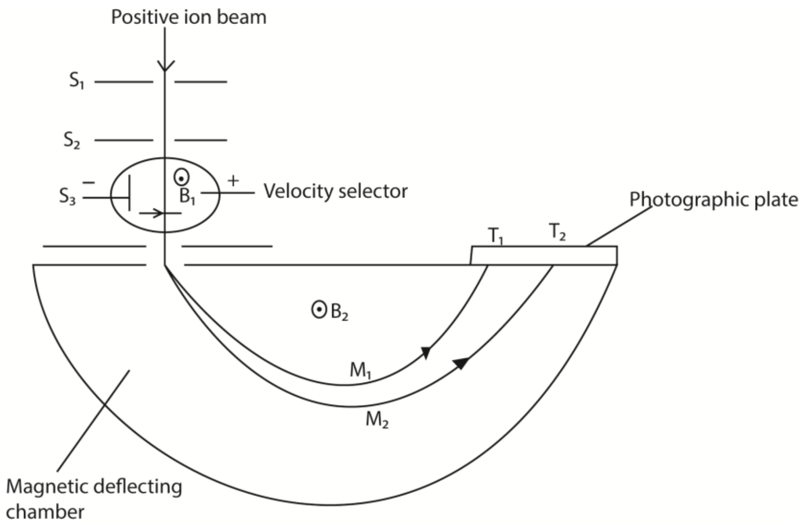
Uses of a Bainbridge mass spectrometer

1. To measure the specific charge of positive rays or ions

2. To separate isotopes of an element

3. To determine the atomic masses of the positive rays or ions

Determination of specific charge using a Bainbridge spectrograph



T_1 and T_2 are tracers on photographic plate, S_1, S_2 and S_3 are slits

Mode of Action

- Positive ions are produced in a discharge tube and admitted as a beam through slits S_1 and S_2.
- The beam then passes between insulated plates P, Q, connected to a battery, which create an electric field of intensity E.
- A uniform magnetic field B_1, perpendicular to E is applied over the region of the plates and all ions, charge e with the same velocity, v given by $B_1ev = Ee$ will then pass undeflected through the plates and through a slit S_3.
- The selected ions are deflected in a circular path of radius r by a uniform perpendicular magnetic field B_2 and an image is produced on a photographic plate as shown.

  In this case

  $$\frac{mv^2}{r} = B_2ev$$

  $$\therefore \frac{m}{e} = \frac{rB_2}{v}$$

  But for the ions selected v = $\frac{E}{B_1}$ from above

  $$\therefore \frac{m}{e} = \frac{rB_2B_1}{E}$$

  Or specific mass-charge ratio or specific charge, $\frac{e}{m} = \frac{E}{rB_2B_1}$,

  $\therefore \frac{m}{e} \propto r$, for given magnetic and electric fields.

Since the ions strike the photographic plate at a distance 2r from the middle of the slit S_3, it follows that the separation of ions carrying the same charge is directly proportional to their mass. Thus a 'linear' mass scale is achieved.

Ionic separation is obtained by

$m_1 = \frac{r_1 B_2 B_1 q}{E}$ and $m_2 = \frac{r_2 B_2 B_1 q}{E}$

$m_1 - m_2 = \frac{B_2 B_1 q}{E}(r_1 - r_2)$

$r_1 - r_2 = \frac{E}{B_2 B_1 q}(m_1 - m_2)$

Where $r_1 - r_2$= ionic separation

Example 16

The magnetic flux density in both fields is 0.4T and the electric field in the velocity selector is 2x10^{4}Vm.

(i) What is the velocity of an ion which goes un deviated through the slit system
$B_1 = B_2 = B$ = 0.4T and E = 2 x 10^{4}Vm
For crossed fields in the velocity selector
$B_1qu = Eq$
∴ u = $\frac{E}{B_1} = \frac{2\ x\ 10^4}{0.4} = 5\ x\ 10^4 ms^{-1}$

(ii) The source is set to produce singly charged ions of magnesium isotopes as Mg – 24 and Mg – 26. Find the distance between the images formed by the isotopes on the photographic plate, assuming the atomic masses of the isotopes are equal to their mass numbers numerically.
In the deflection chamber, $F_m = F_e$
$B_2qu = \frac{mu^2}{r}$, r =$\frac{mu}{B_2q}$, d= $\frac{2mu}{B_2q}$ since r = $\frac{d}{2}$
Also m_1 = 24U and m_2 = 26U
$d_1 = \frac{2m_1u}{B_2q}$ and $d_2 = \frac{2m_2u}{B_2q}$
$d_2 - d_1 = \frac{2u}{B_2q}(m_2 - m_1) = \frac{2\ x\ 5\ x\ 10^4}{0.4\ x\ 1.6\ x 10^{-19}}(26 - 24) x\ 1.67\ x\ 10^{-27}$= 5.22 x 10^{-3}m

(iii) Calculate the ratio of the times the two isotopes take to complete a semi-circle. {Assume 1U = 1.67x10^{-27}kg and e = 1.60x10^{-19}C}

Radius, r =$\frac{mu}{Bq}$ and circumfrence of semicircle = πr = $\frac{\pi mu}{Bq}$

Time taken to complete semi-circle = $\frac{\pi r}{u} = \frac{\pi m}{Bq}$

Ration of time form Mg-24 and Mg-26 = $\frac{24}{26}$

The atom

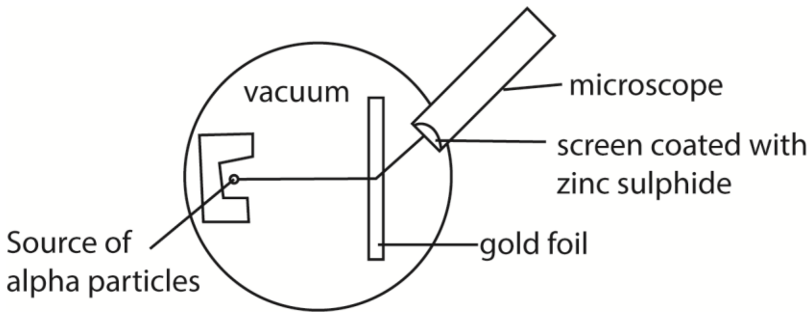
The atom consists of a nucleus containing protons and neutrons (Nucleons) surrounded by orbits carrying electrons. The number of protons in an atom is referred to as **atomic number** whereas the sum of protons and neutrons in the nucleus of an atom is the **mass number** of the atom (nucleon number)

Atoms of the same element with the same number of protons but different number of neutrons are called **isotopes**.

Atoms with the same number of nucleon are called **isobars**.

Rutherford model of an atom

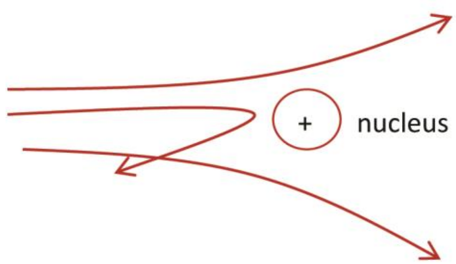
According to Rutherford, an atom consists of a positively charged core (nucleus) which contains most of the mass of the atom and it's being surrounded by orbiting electrons occupying the biggest part of the atom. To confirm this, Rutherford investigated the scattering of alpha particles by a thin foil of heavy metal e.g. gold.



In this experiment, α (alpha particles) from a source are incident onto a thin metal foil and a glass screen coated with zinc sulphide is used to detect the scattered alpha particles which form scintillations (flash of light) as they hit the screen.

It is carried out in a darkened room so that the scintillations can be seen clearly and the apparatus is evacuated to ensure that the particles are able to reach the screen without losing energy.

By rotating the screen about the metal foil and then counting the number of scintillations produced at various positions in equal interval of time, it is observed that majority of α-particles go through undeviated, few of the α-particles are scattered through small angles and very few are deviated by more than 90^0.



The large angle scattering is due to a single encounter between an alpha particle and the intense positive charge (nucleus). Since very few of the particles are scattered through large angles, this confirms that the nucleus occupies a small portion of available space in an atom. This disapproves the plum-pudding model which was initially popular.

Rutherford's model however had some opposition on theoretical ground that the orbit electrons are accelerating thus emitting electromagnetic radiations at the expense of their own energy and consequently they would slow down and spiral into the nucleus.

To resolve this, Bohr assumed that each electron moves in a circular orbit centered on the nucleus and necessary centripetal force is being provided the electrostatic force due to the nucleus and this is concretized by the following Bohr's proposals.

The Bohr Model of the Atom Bohr's postulates

(i) Electrons in atoms exist only in certain discrete orbits and while in these orbits they don't radiate energy.

(ii) Whenever an electron makes a transition (jumps) from one orbit to another of lower energy, a quantum of electromagnetic radiation is given off. The energy of the quantum of radiation emitted is given by: $hf = E_i - E_f$
Where;
E_i is the energy of the electron from the initial orbit,
E_f is the energy of the electron in the final orbit,
f is the frequency of the radiation emitted and h is Planck's constant.

(iii) The angular momentum of an electron in its orbit in an atom is an integral multiple of $\frac{h}{2\pi}$.

Failure

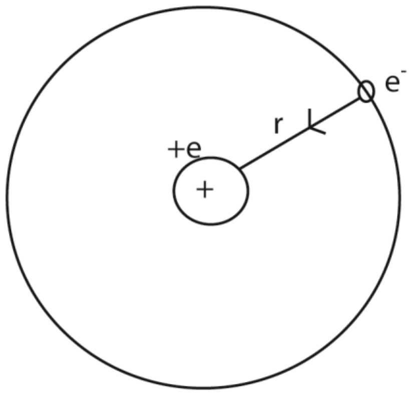
For big atoms $M\omega r = \frac{nh}{2\pi}$ contradits the wave form behavior of electrons

Note: The Bohr atom consists of a massive positively charged nucleus occupying a small space at the center being surrounded by orbiting electron which don't emit electromagnetic radiation as they revolve.

Applications of Bohr's postulatesto the hydrogen atom

Consider the electron in a hydrogen atom to be in a circular orbit of radius r about the nucleus.

An electron has kinetic energy to its motion round the nucleus and potential energy in electrosatic field of the nucleus.



Kinetic energy of electron in the orbit

Centripetal force on electron $= \frac{mu^2}{r}$

Electrostatic force on electron = $\frac{ee}{4\pi\varepsilon_0 r^2}$

Centripetal force = electrostatic force

$$\frac{mu^2}{r} = \frac{ee}{4\pi\varepsilon_0 r^2}$$

$$mu^2 = \frac{e^2}{4\pi\varepsilon_0 r^2} \text{ .... (i)}$$

Multiply (i) by ½

$$\frac{1}{2}mu^2 = \frac{e^2}{8\pi\varepsilon_0 r^2}$$

$$\therefore K.E = \frac{e^2}{8\pi\varepsilon_0 r^2} \text{ ....(ii)}$$

Potential energy of an electron in orbit

The potential due to the charge e on the nucleus at a distance r is given by

V = $\frac{+e}{4\pi\varepsilon_0 r}$

P.E of electron = qV = $\frac{-e.e}{4\pi\varepsilon_0 r} = \frac{-e^2}{4\pi\varepsilon_0 r}$ .... (iii)

Total energy of electron in orbit E = P.E + K.E

$$E = \frac{-e^2}{4\pi\varepsilon_0 r} + \frac{e^2}{8\pi\varepsilon_0 r^2} = \frac{-e^2}{8\pi\varepsilon_0 r^2} \text{ .... (iv)}$$

The radii of the Bohr orbits

Angular momentum of the electron in an orbit of radius r is given by (mu) x r

i.e. the product of linear momentum and radius

Applying Bohr's postulates regarding angular momentum mur $= \frac{nh}{2\pi}$ …………………. (v)

(v) Squared

$m^2u^2r^2 = \frac{n^2h^2}{4\pi^2}$ ……….. (vi)

(i) and (vi)

r = $\frac{n^2h^2\varepsilon_0}{m\pi e^2}$

or

$r_n = \frac{n^2h^2\varepsilon_0}{m\pi e^2}$ where n = 1, 2, 3, ……………….

This gives the radii of the allowed Bohr orbits

The allowed electron energies are obtaned by substituting r_n for r in equation (iv)

$$E_n = \frac{-me^4}{8\pi\varepsilon_0 h^2 n^2} \quad n = 1, 2, 3,$$

Bohr's assumptions

1. Each electron moves in a circular orbit with the nucleus as its center.

2. The necessary centripetal force is provided by the electrostatic force of attraction between the positively charged nucleus and the negatively charged electron.

Bohr's failures

1. It can only explain spectra for simpler atoms with few electrons such as hydrogen

2. It cannot explain fine structure of spectral lines of hydrogen

3. His model assumes that the electron orbits are circular yet they are elliptical.

Note:

(i) The energy of the electron is always negative. This means that the work has to be done to remove the electron from infinity where it is considered to have zero energy. The electron is thus bound to the atom.

(ii) When r_n is increased, n also increases and the energy E_n becomes less negative.

(iii) The lowest energy level or energy state occurs when n = 1 and is referred to as the ground state. The other energy levels (energy state) are called excited energy states.

(iv) A transition of an electron from energy level n1 to n2 will lead to a radiation of energy hf such that $E_1 - E_2 = hf$

$$hf = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_2} - \frac{1}{n_1}\right)$$
$$f = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_2} - \frac{1}{n_1}\right)$$
$$f = R\left(\frac{1}{n_2} - \frac{1}{n_1}\right)$$ where R is the Rydberg's constant

(v) The energy of the stationary/ground state with least energy (n = 1) is

$$E_1 = \frac{-me^4}{8\pi\varepsilon_0 h^2} = \frac{-9\,x\,10^{-31}\left(1.6\,x\,10^{-19}\right)^2}{8(8.85\,x\,10^{-12})^2(6.6\,x\,10^{-34})^2} = -2.18 \text{ x } 10^{-18}\text{J} = -13.6\text{eV}$$

The energies of the other stationary states can be expressedas

$$E_n = -\frac{13.6}{n^2}ev$$

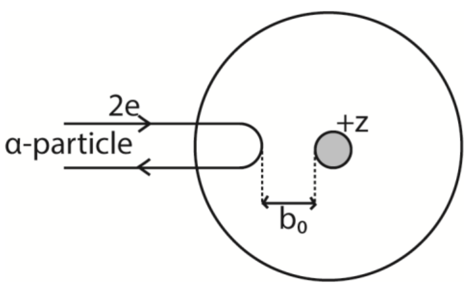
Approximate value are

E_2 = -3.39eV, E_3 = - 1.59eV, E_4 = -0.85eV

(vi) Bohr atom is defiened as one whose center is the nucleus surrounded by electrons moving in definite orbitals.

Distance of closest approach

Consider an alpha particle with charge, 2e incident directly towards the nucleus of charge +ze.



An alpha particle approaching directly the nucleus is slowed down and comes to rest a distance, b_0 from the nucleus and then repelled back.

The kinetic energy possessed by an approaching alpha particle is given by ½ mv^2

The electrostatic potential energy of alpha particle and the nucleus at closest distance of approach is given by

P.E (electrostatic) = $\frac{Q_1 Q_2}{4\pi\varepsilon_0 b_0}$

$$= \frac{(2e)(ze)}{4\pi\varepsilon_0 b_0} = \frac{2ze^2}{4\pi\varepsilon_0 b_0}$$

At the distance of closest approach

K.E = P.E

$\frac{1}{2}mv^2 = \frac{2ze^2}{4\pi\varepsilon_0 b_0}$

$b_0 = \frac{ze^2}{\pi\varepsilon_0 mv^2}\ or\ \frac{ze^2}{2\pi\varepsilon_0 K.E}$

Where z= atomic number of the nucleus

e = electronic charge

m = mass of approaching nucleus

v = speed of the approaching nucleus

Example 17

A beam of α-particle of 4.2MeV is incident to a nucleus of a gold atom. Calculate the distance of closest approach (Z = 79)

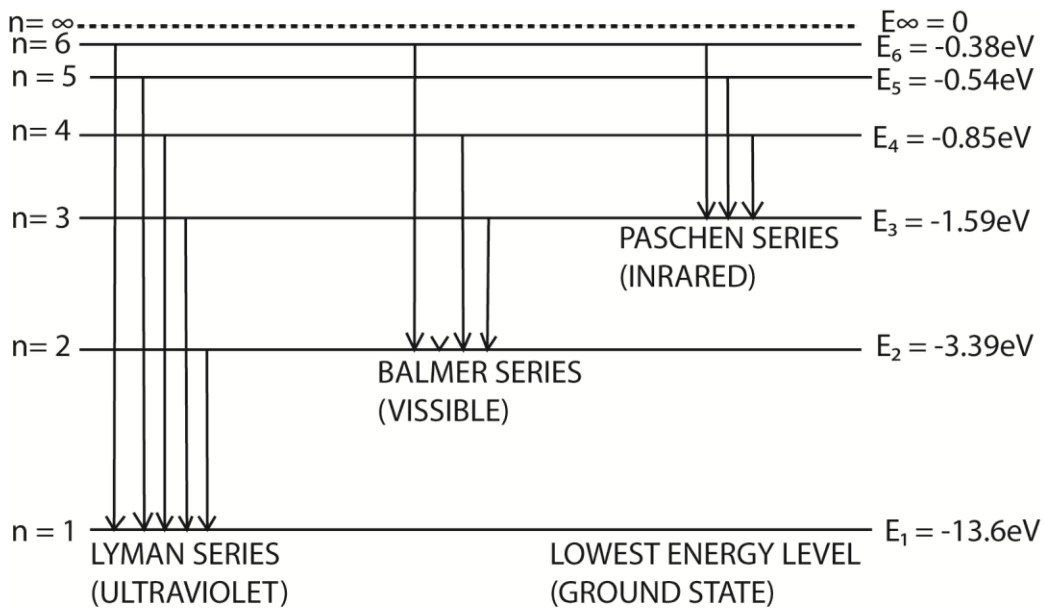
$b_0 = \frac{ze^2}{2\pi\varepsilon_0 K.E} = \frac{79\ x\ (1.6\ x\ 10^{-19})^2}{2\pi\ x\ 8.85\ x\ 10^{-12}\ x\ 6.72\ x\ 10^{-13}}$ = 5.412 x 10^{-14}m

Energy Level

- According to Bohr's model of an atom, electrons are arranged in permitted (allowed) orbits of definite amount of energy. These orbits are also known as energy levels of an atom
- The energies of the electrons in an atom have only certain values called energy levels of the atom.
- All atoms of a given element have the same set of energy levels and these are characteristic of the element i.e. they are different from those of other elements.
- Energy levels are calculated using Bohr model and expressed in eV
- The lowest energy level of an atom is known as ground state.
- Electrons have least energy in the ground state.
- The atom becomes ionized when an electron receives energy to just exceed its highest energy level and leave the atom.
- The highest energy level given the value zero and lower energy levels are negative.

Energy level in the hydrogen atom

The spectrum of atomic hydrogen contains distinct groups of lines. The three major ones include; Lyman series, the Balmer series and the Paschen series



- The spectral lines of hydrogen are experimental evidence for the existence of discrete or separate energy levels of the hydrogen atom.
- Transition involving LYMAN SERIES involve high energy changes. Those in PASCHEN SERIES involve lowest energy changes.
- Lyman series involve the transition to the energy state n_1 and the resulting radiation emitted is the Ultra-Violet.
- Balmer series involve the transition to the energy state n_2 and the resulting radiation emitted is the visible spectrum.
- Paschen series involve the transition to the energy state n_3 and the resulting radiation emitted is the Infra-red.
- -The energy required to remove the electron in the ground state to infinity is the ionization energy

  Thus ionization energy, $E_\infty = 0 - E_1 = 0 - (-13.6eV)$ = 13.6eV = 2.176 x 10^{-18}J

Transitions

- A transition of an electron at a higher energy level E_x to a lower energy level E_y results in loss of energy given by: $(E_x – E_y)$ = hf; Where h is Planck's constant, f is the frequency of the electromagnetic radiation.
- Sometimes an electron may pass through an intermediate state E_m to the final energy state. It emits frequencies f_1 and f_2 given by $E_x – E_m = hf_1$ and $E_m – E_y = hf_2$
- When an electron absorbs energy, it jumps from a lower energy level to a higher energy level. The atom is then said to be excited. Such an atom is unstable and to gain stability, the electron falls back to its original energy level while releasing the energy in a form of radiations (light).

Note:

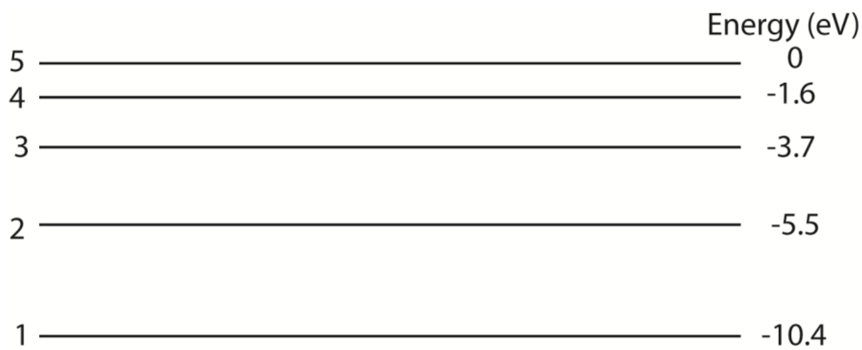
1) Excitation Energy: This is the energy required to raise an atom from its ground state to an excited state.

2) Ionization Potential: This is the potential required to enable the electron to escape completely from the atom.

3) Ionization Energy: this the energy required by an electron to escape completely from the atom

Example 18

Some of the energy levels of mercury are shown in the diagram below. Level 1 is the ground state level occupied by the electrons in an unexcited atom



(i) Calculate the ionization energy of mercury atom in Joules

Ionization energy = $E_I = E_\infty - E_1$

= 0 – (-10.4)

= 10.4eV

= 10.4 x 1.6 x 10^{-19}

= 1.66 x 10^{-18}J

(ii) (ii) Calculate the wavelength of the radiation emitted when an electron moves from level 4 to level 2.

Solution

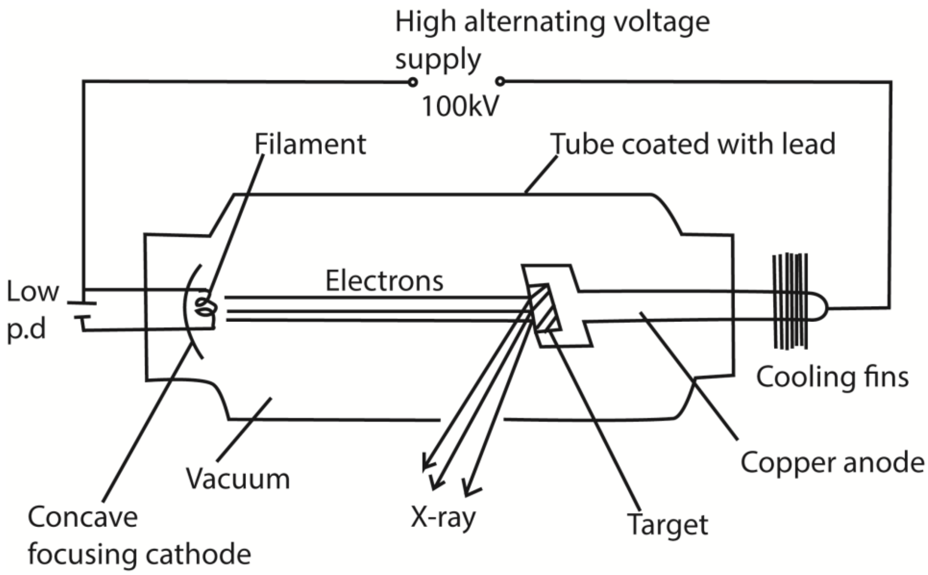
$E_4 - E_2 = \frac{hc}{\lambda}$

$\lambda = \frac{6.63\ x\ 10^{-34}\ x\ 3\ x\ 10^{8}}{-(1.6 - 5.5) x\ 1.6\ x\ 10^{-19}}$ = 3.2 x 10^{-7}m

X-Rays

X- Rays are electromagnetic radiations of short wavelength produced when fast moving electrons are stopped by heavy metal target.

Production of X – Rays



Mode of operation

- The filament is heated by a low voltage supply and the electrons are emitted by thermionic emission.
- The concave focusing cathode focuses the electrons from the filament onto the target.
- These electrons are accelerated towards the anode by the high voltage between the filament and the Anode.
- When the electrons (cathode rays) strike the metal target, about only 1% their kinetic energy is converted to X-rays and the 99% of their kinetic energy is converted to heat, which is conducted away by the cooling fins.

Note.

i) The target is made of a high melting point metal.

ii) ii) The X-ray tube is covered by a lead shield with a small window for the X-rays to prevent the leakage of the X-rays.

Intensity of X-rays (Quantity or number of X-rays)

- The intensity of X- rays in an X – ray tube is proportional to the number of electrons colliding with the target.
- The number of electrons produced at the cathode depends on the filament current supply.
- The greater the heating current, the greater the number of electrons produced and hence more x- rays are produced.
- Therefore the intensity of X- rays is controlled by the filament current.

Penetration of X – rays (quality)

- Penetration power of X-rays depends on the kinetic energy of the electrons striking the target.
- The higher the accelerating voltage, the faster the electrons produced.

- Faster electrons possess higher kinetic energy and shorter wavelength x-rays of greater penetration power are produced.
- Hence penetrating power of X-rays is determined by the accelerating Voltage across the tube.

Types of X-rays

There are two types of X-rays, namely: Hard X-rays and soft X- rays

Hard X-rays:

- They are produced when a high p.d is applied across the tube.
- They have very short wave lengths
- They have a high penetrating power. This is because they have very short wavelengths

Soft X-rays:

- They are produced by electrons moving at relatively lower velocities than those produced by hard X –rays.
- They have longer wavelengths.
- They have a low penetration power compared to hard x-rays. This is because of their long wavelengths

Note:

- Hard X-rays can penetrate flesh but are absorbed by bones, they are therefore used to study bone fractures.
- Soft X-rays are used to show malignant growths since they only penetrate soft flesh. They are absorbed by such growths.

Properties of X –rays

1) They travel in a straight line at a speed of light in vacuum
2) They are not deflected by both magnetic and electric fields. This indicates that they carry no charge.
3) They penetrate all matter to some extent. Penetration is least in materials with high density and atomic number e.g. lead.
4) They ionize gases through which they pass.
5) They affect photographic plates.
6) They cause fluorescence in some materials.
7) They cause photoelectric emission
8) They are diffracted by crystals leading to an interference pattern.

Uses of X-rays

1. Structural analysis, stresses, fractures in solids, castings and welded joints can be analyzed by examining X-ray photograph.
2. Crystallography; Orientation and identification of minerals by analysis of diffraction patterns using Bragg's law.
3. Medical uses;

i) Analytical uses. These include location of fractures, cancer and tumor/defective tissue absorbs x-rays differently from normal tissue.

ii) Therapeutics use for destroying cancerous cells and tumors. 4. Detection of fire arms at international airports

Health hazards caused by x-rays:

- Destroy living cells in our bodies especially hard X-rays.
- Cause Gene mutation (genetic changes in our bodies).
- Cause damage of our eye sight and blood. Produce deep skin burns.

NOTE: It's highly important to remember that each time you are exposed to X-rays, your health is also at risk yet we cannot live without them

Safety precautions:

- Avoid unnecessary exposure to X-rays.
- When exposure is necessary, keep it as short as possible.
- X-ray beams should ONLY be restricted to the body part being investigated.
- A worker should wear a shielding jacket with a layer of Lead.
- Exposure should be avoided for unborn babies and very young children.

Example 19

In an X-ray tube, the current through the tube is0.1mA and accelerating p.d 1.5kV. Calculate the:

(i) The number of electrons striking the anode per second

I = ne where n is the number of electrons striking the anode per second

n = $\frac{0.1 \, x \, 10^{-3}}{1.6 \, x \, 10^{19}}$ = 6.25 x 10^{14}

(ii) The speed of electron striking the anode

$\frac{1}{2}mv^2 = eV$

$v = \sqrt{\frac{2 \, x \, 1.6 \, x \, 10^{-19} \, x \, 1.5 \, x \, 10^{3}}{9.11 \, x \, 10^{-31}}}$ 2.295 x $10^7 ms^{-1}$

(iii) The rate at which cooling fluid at 10^0 must be circulated through the tube if the anode is to be maintained at 35^0C.

[Assume all electrical energy is converted into heat energy and S.H.C of fluid is $2000Jkg^{-1}K^{-1}$]

IVt = mc ($\theta_2 - \theta_1$)

IV = $\frac{m}{t}(\theta_2 - \theta_1)$

= KC($\theta_2 - \theta_1$) where K is the rate of flow

K = $\frac{0.1 \, x \, 1.5 \, x \, 10^3}{2000 \, x \, (35-10)}$ = 3 x $10^{-3} kgs^{-1}$

Example 20

In an X-ray tube, 90% of the electrical power supplied is dissipated as heat. If the accelerating potential difference across the tube is 75kV and 742.5W is dissipated as heat, calculate the:

(i) Current in the tube

P = IV; V = 75kV

90% of IV= heat lost

$\frac{90}{100}\ x\ I\ x\ 75\ x\ 10^3\ =\ 742.5$
$I = 0.011A$

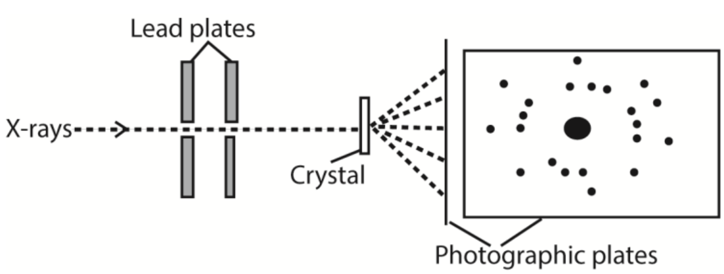
(ii) Number of electrons arriving at the target per second
I = ne where n is the number of electrons per second
n = $\frac{0.011}{1.6\ x\ 10^{-19}}$ = 6.875 x 10^{16}

Example 21
The current in a water-cooled X-ray tube operating at 60kV is 30mA. 99% of the energy supplied to the tube is converted into heat which is removed by water at a rate of 0.06kgs^{-1}. Calculate the:
(i) Number of electron hitting the target per second
Number of electrons per second = $\frac{I}{e} = \frac{30\ x\ 10^{-3}}{1.6\ x\ 10^{-19}}$ = 1.875 x 10^{17}electrons per second
(ii) Rate at which energy is being supplied to the tube
Power = IV = 30 x 10^{-3} x 60 x 10^3 = 1800W
(iii) Rate of change in temperature of cooling water
99%IV = heat lost per second
$\frac{99}{100}IV = m'c\theta$ where m' is rate of flow kgs^{-1}
$\frac{99}{100}\ x\ 1800 = 0.06\ x\ 4200\ x\ \theta$
$\theta = 7.07Ks^{-1}$

X-ray diffraction

The wave nature of X-rays can be confirmed by their diffraction with crystals

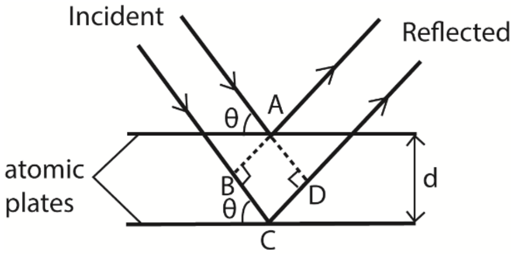
Laue's experiment:



- After long exposure of the crystal to the x-rays, the photographic plate is developed and printed.
- A regular pattern of dark sports called Laue spots is observed around a central dark image.
- The pattern is due to the X-rays which have been scattered by interaction of the X-rays with the electrons in the atoms of the crystal.
- The regularity of the Laue spots implies that the atoms in a crystal are arranged in a regular pattern.

Bragg's law

- A parallel beam of monochromatic X-rays incident on a crystal is reflected from successive atomic planes and super-imposed, forming an interference pattern.



For constructive interference to occur, the path difference is equal to the whole number of wavelength

Thus BC + CD = nλ

⇨ dsinθ + dsinθ = nλ

or 2dsinθ = nλ where n = 1, 2, 3, 4 …

Example 22

A second order diffraction mage is obtained by reflection of X-rays at atomic planes of a crystal in sodium chloride at glancing angle of 11^0. Calculate the atomic spacing of the planes if the wavelength of X-rays is 4×10^{-11}m.

From 2dsinθ = nλ

$d = \frac{2 \; x \; 4 \; x \; 10^{-11}}{2 \sin 11}$ = 2.096×10^{-10}m

Example 23

X-ray of wavelength 1.55×10^{-10}m are incident on a copper crystal of atomic spacing 4.25×10^{-10}m

(i) Calculate the smallest angle at which radiation will be first reflected.
From 2dsinθ = nλ
$\sin \theta = \frac{1 \; x \; 1.55 \; x \; 10^{-10}}{2 \; x \; 4.25 \; x \; 10^{-10}}$
$\theta = 10.5^0$

(ii) If the temperature of the crystal is increased by 600^0, calculate the change in the angle that will be obtained. [the coefficient of linear expansion of copper is $1.7 \times 10^{-5}K^{-1}$]

From $C_\theta = C_0(1 + \alpha\theta)$
$d_\theta = 4.25 \times 10^{-10}(1 + 1.7 \times 10^{-5} \times 600) = 4.29335 \times 10^{-10}$m
$\sin \theta' = \frac{1 \; x \; 1.55 \; x \; 10^{-10}}{2 \; x \; 4.29335 \; x \; 10^{-10}}$
$\theta = 10.4^0$
change in angle = 10.5 – 10.4 = 0.1^0

Example 20

A monochromatic beam of X-rays of wavelength 2×10^{-10}m is incident on a set of cubic planes in potassium chloride crystal. First order diffraction is observed at glancing angle 18.5^0.

Calculate

(i) The inter-atomic spacing on potassium chloride.

From 2dsinθ = nλ

$d = \frac{1 \, x \, 2 \, x \, 10^{-10}}{2 \sin 18.5}$ = 3.152 x 10^{-10}m

(ii) The density of potassium chloride if the RFM is 75.5grams

For the two ions in KCl

V = $2d^3$ = 2 x $(3.152 \text{ x } 10^{-10})^3$ = 6.263 x $10^{-29}$$m^3$

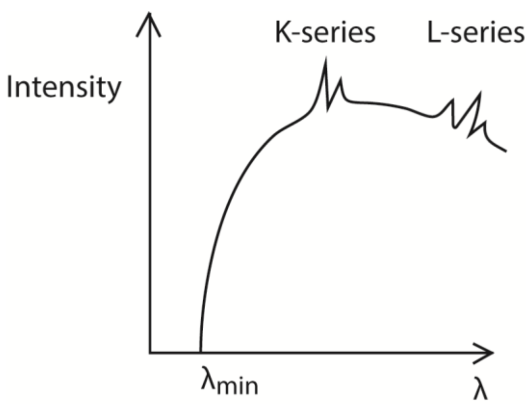
RFM KCl = 75.5

6.02 x 10^{23} molecules of KCl weigh 75.5g

1 molecule weighs $\frac{75.5}{6.02 \, x \, 10^{23}}$ = 1.254 x 10^{-22}g

ρ = $\frac{m}{V} = \frac{1.254 \, x \, 10^{-22}}{6.263 \, x \, 10^{-29}}$ = 2002.2kgm^{-3}.

X-rays Emission spectrum



The spectrum consists of two major components, i.e. the continuous (background) spectrum and the very sharp line spectrum superimposed onto the background spectrum.

The continuous spectrum is produced when electrons make multiple collisions with the target atoms in which they are decelerated. At each deceleration, X-rays of differing wavelength are produced.

The shortest Wavelength X-rays are produced when electrons lose all their energy as X-ray photon in a single encounter with the target atoms. The wavelength of the X-rays at this point is known as the cut off wavelength. At cut off wavelength, energy in an X-ray photon equals kinetic energy of the electron; i.e. hf = eV or $\frac{hc}{\lambda_{max}} = eV$ where V = p.d

The line spectrum

At high tube voltages, the bombarding electrons penetrate deep into the target atoms and knock out electrons from inner shell. The knocked out electrons occupy vacant spaces in higher unfilled shells putting the atom in excited state and making them unstable.

Transition of an electron from higher to lower energy levels results in an emission of X-ray photon of energy equal to energy difference between the energy levels.

If the transition ends in the K-shell, it produces K-series and if the transition ends in L-shell. It produces L-series.

Radioactivity

This is the spontaneous disintegration of unstable atoms with emission of particles like alpha, beta particles and gamma radiations.

The nuclei of some elements like uranium, thorium are unstable undergo radioactive decay in order to gain stability.

The three types of radiations can be identified by:

1) Their different penetrating powers/abilities

2) Their ionizing powers

3) Their behavior in electric and magnetic fields.

Alpha – particles, ^4_2He

- They are the least penetrating with a range of a few centimeters in air and can be stopped by paper.
- They produce intense ionization in any gases through which they pass.
- They are deflected by both electric and magnetic fields.
- Their direction and size of deflection suggests that:
  i) They are positively charged
  ii) They are relatively heavier particles.
- Alpha particles are therefore a Helium nuclei containing 2 protons and 2 neutron

Beta – particles

- They are more penetrating than the alpha particles with a range of several centimeters in air and a few millimeters in aluminum.
- They are less ionizing than the alpha particles.
- They are more easily deflected than the alpha – particles, and their size and direction of deflection suggest that:
  i) They are negatively charged
  ii) They have a very small mass

Gamma rays

- They are highly penetrating
- They ionize gases to a very small extent
- They are not deflected by both the magnetic and electric fields, indicating that they are uncharged.

Rules governing radioactivity

1. When a radioactive substance decays by emission of alpha particle, its atomic number A reduces by 2 and it mass number Z reduces by 4
   i.e. ${}_{A}^{Z}X \rightarrow {}_{A-2}^{Z-4}Xy + {}_{2}^{4}He$
2. When a radioactive substance decays by emission of beta particle, its atomic number A increases by 1 and it mass number Z remains constant
   . ${}_{A}^{Z}X \rightarrow {}_{A+1}^{Z}Xy + {}_{-1}^{0}e$
3. When a radioactive substance decays by emission of gamma rays, both its atomic number A and it mass number Z remains constant
   ${}_{A}^{Z}X \rightarrow {}_{A}^{Z}Y + \gamma$

The decay law

It states that the rate of disintegration of the nuclei in a given time is propotional to the number of atoms present

Rate of decay,R = $-\lambda \frac{dN}{dt}$

where N- number of atoms present, t = time, λ is a constant and negative because the number of atoms are reducing

The decay law can also be xpressed as

$N = N_0e^{-\lambda t}$

where No is the intial number of disintegrating atoms.

The decay constant is the fractional number of atoms that are disintegrating per second

Half-life ($t_{½}$) is the time taken for the number of atoms in a radioactive element to reduce to half the original value.

From $N = N_0e^{-\lambda t}$

In $\frac{N_0}{N}$ = λt

At t = t ½ ; N = $\frac{N_0}{2}$

⇨ In $\frac{N_0}{N/2}$ = λt½

⇨ t½ = $\frac{In\ 2}{\lambda} = \frac{0.693}{\lambda}$

Activity is the rate of dsintegration of a radioactive substance = λN

Example 25

A sample of radioactive material initially contains 10^{18} atoms. If the half-life of the material is 2 days, calculate the

(i) number of atoms remaining after 5days

$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{2} = 0.3465s^{-1}$

N = $10^{18}e^{-0.3467 \, x \, 5}$ = 1.7684 x 10^{17}

(ii) percentage that decayed after 5days

Number of decayed atoms = $N_0 - N$

Percentage decayed = $\frac{N_0 - N}{N_0} \, x \, 100\%$ = $\frac{10^{18} - 1.7684 \, x \, 10^{17}}{10^{18}}$x100% = 82.32%

(iii) activity of the sample after 5days

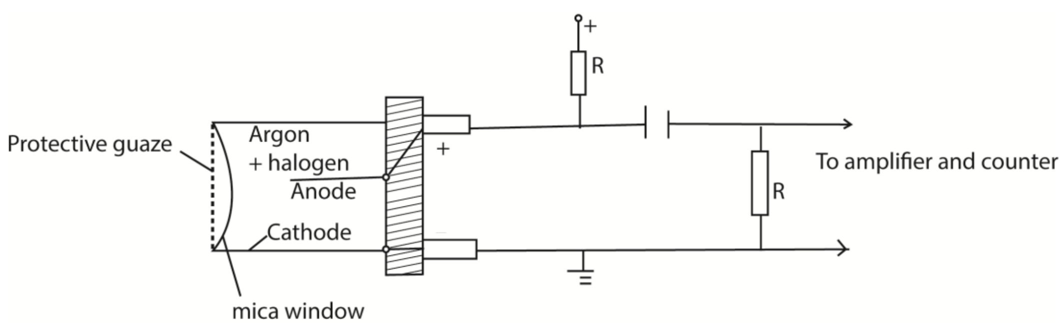
Activity, A = λN = 0.3465 x 1.7684 x 10^{17} = 6.127506 cx 10^{16}

Radioactive detector

A. Geiger Muller Tube (GMT)

The GMT is used to detect the presence of X- rays, Gamma rays, beta particles and if the window of the tube used is very thin, it detects even alpha particles.

Structure



The thin mica window allows the passage and detection of the weak penetrating alpha particles. The GM tube is first evacuated then filled with Neon, Argon plus Halogen gas which is used as a quenching agent.

Mode of operation

Mode of operation

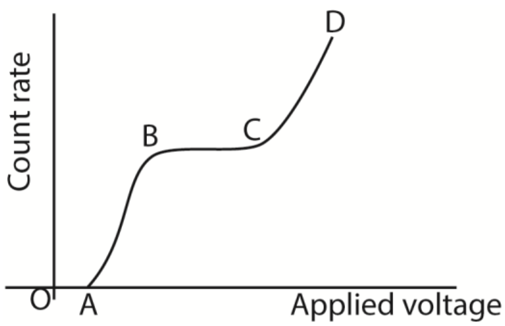
- When an ionizing particle enters the tube through the window, argon atoms are ionized.
- The electrons move to the anode while the positive ions drift to the cathode.
- A discharge occurs and the current flows in the external circuit.
- A p.d is obtained across a large resistance, R which is amplified and passed to the scale.
- The magnitude of the pulse registered gives the extent to which ionization occurred.

Note:

- The anode wire must be very thin so that the charge on it produces an intense electric field close to its surface.
- This electric field is used to accelerate the electrons towards it from the cathode.

Characteristics of a GMT

The graph below is obtained when the counter rate is plotted against the operating voltage.



OA – the operating voltage is not enough to attract the ions to the respective electrodes and hence the counter registers no reading. This voltage (i.e. at A) is called the threshold voltage.

AB – the applied p.d not enough to attract all electrons; hence increasing the p.d increases the number of electrons being attracted and hence increase in counter rate.

BC – here the count rate is constant. This is called the plateau region.

- Between BC, all the negative ions are able to reach the anode because the operating voltage is large enough to attract them.

- Full avalanche is obtained along the entire length of the anode.

- Here the tube is said to be operating normally.

CD: - The count rate increases rapidly because the quenching process becomes ineffective and eventually a continuous discharge occurs which might damage the tube.

Definitions.

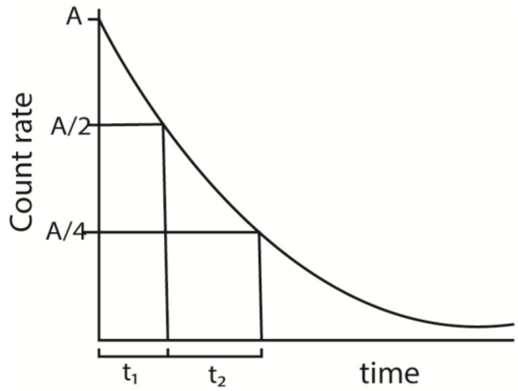
Dead time. This is the time taken by the positive ions to move from the anode to the cathode. During this time the tube is insensitive to the arrival of further ionizing particles.

Recovery time. This is the second period of insensitivity. During this period, pulses are produced but not large enough to be detected. In this time, argon ions are being neutralized by the quenching gas before they reach the anode.

Threshold voltage is the voltage below which there is no sufficient gas amplification to produce pulse high enough to be detected

Experiment to determine a half-life of radioactive substance using GM- tube

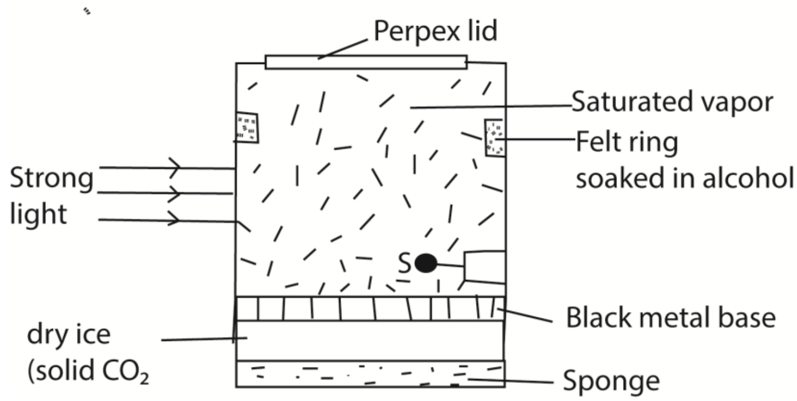
- Switch on the GM-tube, note and record the background count rate, A.
- Place a source of ionizing radiation near the GM-window.
- Note and record the count rate recorded the count rate at equal intervals.
- For each count rate recorded subtract the background count rate to get the true rate.
- Plot a graph of the count rate against time.



Find time t_1 taken for the activity to reduce to A/2 and t_2 taken for activity to reduce to A/4 from A/2

Half-life = $\frac{1}{2}(t_1 + t_2)$

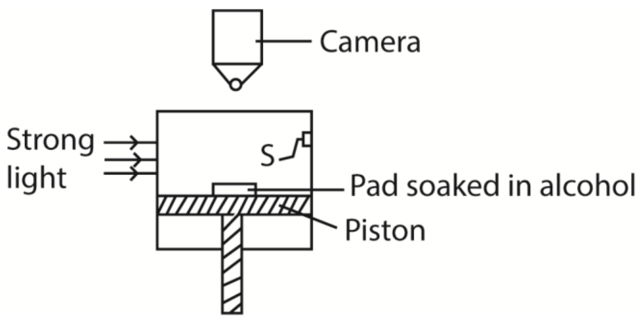
B. The diffusion cloud chamber



- The base of the chamber is maintained at low temperature, about -80^{0}c by the solid carbon dioxide while the top of the chamber is at room temperature, and so there is a temperature gradient between the top and the bottom of the chamber.
- The air at the top of the chamber is saturated with alcohol vapor from the felt ring. This vapor continuously diffuse downwards into the cooler regions so that the air at the chamber is super saturated with alcohol vapor.
- Radiations from the radioactive source S cause the ionization of the vapor.
- The ionizations from the radioactive source S cause condensation of the vapor on the ions formed, hence the path of the ionizing radiations are traced by series of small droplets of condensation.

- The thickness and length of the path indicate the extent to which ionization has taken place.
- Alpha particles produce short, thick, continuous straight tracks
- Beta particles which are less massive produce longer, thin but straggly paths owing to collisions with gas molecules
- Gamma radiations are uncharged and for ionization to take place, it must collide with an atom and eject an electron which then ionizes the vapor.

C. The Wilson cloud chamber



Mode of action

When the piston is quickly moved, the air in the chamber is saturated with alcohol vapour undergoes an adiabatic expansion and it cools.

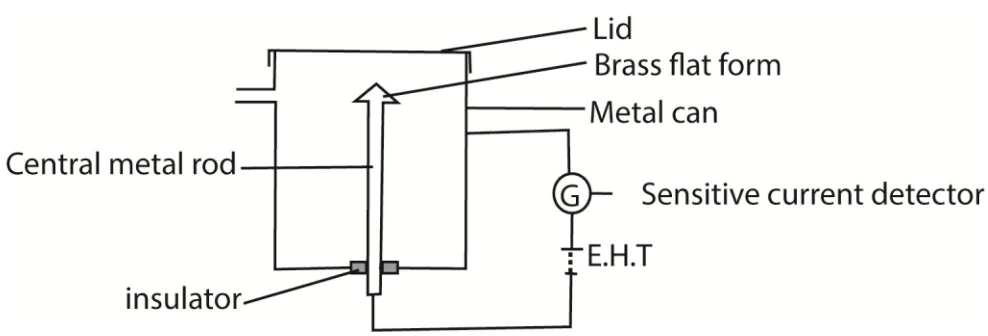
The dust particles are carried away leaving behind air which is dust free. This is the subjected to controlled expansion making it super saturated.

It is then simultaneously subjected to ionizing radiation from a source, S. the vapour condenses on the ions formed to form water droplets around the ions

These are then illuminated and photographed by the camera.

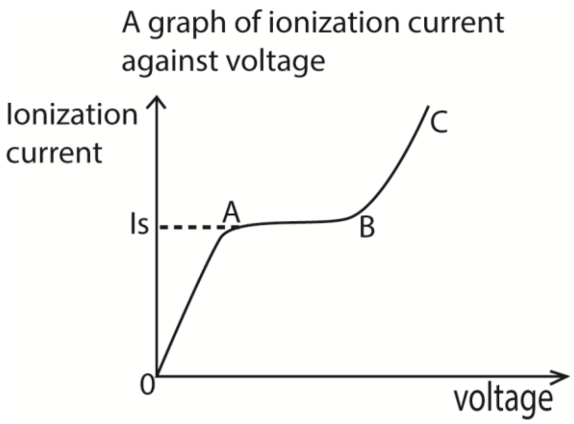
The nature of the path formed reveals the type of ionizing agent.

D. Ionizing Chamber



- A radiation source on the brass flat form causes ionization of air in the chamber producing electrons and positive ions.
- The electrons move to the metal can and positive ions drift to the central metal rod.

- Movement of the ions to the electrodes causes discharge and current pulse flows in external circuit.
- The current sensitive detector detects current.
- The magnitude of current detected shows the extent to which ionization takes place.



Region OA:

Current detected increases gradually but p.d is not large enough to prevent recombination of the ions.

Region AB. (saturation region)

Current is almost constant, all ions reach the electrode before recombination but there is no secondary ionization.

Region BC (gas amplification)

Current increases rapidly for small increase (change) in p.d because secondary ionization takes place due to primary ions being produced. This implies many ion pairs, thus a larger current detected.

Artificial radiations

These can be produced by

- Bombarding a nucleus of stable element with neutrons in a nuclear reactor
- Bombarding a nucleus of stable element with a charged particle such as alpha or beta particles.

Uses of Radioistopes and radioactivity

In industry

- Sterilization of food
- Detecting leakages in pipes
- Determining thickness of paper
- Determining the rate of wear

In medicine

- Treatment of cance
- Tracer of disease

- Sterilizing medical equipment

Health hazards

- May cause cancers
- Eye damage
- Cause sterility
- Cause mutation

Unified Atomic Mass unit U

Definition: the Unified Atomic Mass Unit (U) is one – twelfth the mass of one atom of carbon – 12 ($^{12}_{6}c$)

Derivation

1mole of a substance contains 6.02 x 10^{23}atoms

12g of carbon-12 contains 6.02 x 10^{23} atoms

Mass of 1 atom of carbon-12 = $\frac{12\ x\ 10^{-3}}{6.02\ x\ 10^{23}}$ kg

= 12U

$\therefore$1U = 1.66 x 10^{-27}kg

But 1 kg change in mass produces 9 x 10^{16} joules and 1 Mev = 1.6 x 10^{-13}joules

$$1U = \frac{1.66\ x\ 10^{-27}\ x\ 9\ x\ 10^{16}}{1.6\ x\ 10^{-13}} = 931MeV$$

Binding energy (B)

The mass of the nucleus of atom is always less than the total mass of its constituent nucleons (protons and neutrons). The difference in mass is called the mass defect.

Mass defect = (mass of nucleons) - mass of the nucleons mass of the nucleus

The reduction in mass is because when the nucleons are combining to form the nucleus, some of the mass is released as energy in the form of gamma rays.

In order to break the nucleus and separate the nucleons, the same amount of energy which was released has to be supplied to the nucleus. This is called the Binding energy.

Definition:

Binding Energy is the minimum energy required to break the nucleus into its constituent particles and completely separate them from each other.

Binding energy = (mass defect in kg) X (speed of light)^2

Example 26

The mass of lithium $^{7}_{3}Li$ is 7.01818U. Calculate

(i) The binding energy of lithium atom

Solution
Number of protons = 3
Number of neutrons = 7-3 = 4
Mass of nucleons = (3 x 1.0081 + 4 x 1.009) = 7.067U
Mass defect = mass of nucleons – mass of nucleus
= 7.060 – 7.01818
= 0.04252U
But 1U= 931eV
Binding energy = 0.04252 x 931 = 39.58612eV

(ii) The binding energy per nucleon of lithium atom
Given mass of proton = 1.0081U
Mass of neutron = 1.009U
Mass of electron = 0.00055U
1U = 931eV

Binding energy per nucleon = $\frac{Binding\ energy}{mass\ number}$
$= \frac{39.58612\text{eV}}{7}$
$= 5.65516eV$

Example 27

Given that the mass of $^{210}_{84}Po$ = 209.992U, $^{206}_{82}Pb$ = 205.964U, $^{4}_{2}He$ = 4.02U and 1U = 931eV;

(i) State whether it is possible for $^{210}_{84}Po$ to undergo alpha decay.
$^{210}_{84}Po \rightarrow$ $^{206}_{82}Pb$ + $^{4}_{2}He$

Total mass on RHS = 203.964 + 4.02 = 209.984U

Since there is a loss in mass, the reaction is possible and the loss in mass in mass is the energy released.

(ii) Calculate the mass defect of the reaction
Mass defect (loss in ass = 209.992 – 209.984 = 0.008U
(iii) Find the total energy released in the above reaction
0.008U = 0.008 x 931MeV =7.448MeV

Example 28
When a fast moving neutron hits uranium, $^{235}_{92}U$ the nucleus breaks up into$^{95}_{57}Mo$,$^{139}_{57}La$, 2neutrons and 7electrons.
Calculate the energy released by 10grams of uranium in the reaction
[$^{235}_{92}U$ = 235.044U, $^{95}_{57}Mo$ = 94.906U, $^{139}_{57}La$ = 138.906U, $^{1}_{0}n$ = 1.009U, $^{0}_{-1}e$ = 0.005U, 1U = 1.66 x 10-27kgJ]
Solution
$^{235}_{92}U$ + $^{1}_{0}n \rightarrow$ $^{95}_{57}Mo$ + $^{139}_{57}La$ $2^{1}_{0}n$ + $7\,^{0}_{-1}e$

Mass defect = (235.044 + 1.009) – (94.906 + 138.906 + 2 x 1.009 + 7 x 0.005)

= 0.188U

= $(0.188\ x\ 1.66\ x\ 10^{-27})$kg

= 3.1208 x 10^{-28}kg

From E = mc^2

Energy released = 3.1208 x 10^{-28} x (3 x $10^8)^2$

= 2.81 x 10^{-11}J

1mole mole $^{235}_{92}U$ contains 6.02 x 10^{23} atoms

235g contain 6.02 x 10^{23} atoms

10g contain $\frac{6.02\ x\ 10^{23}}{235}$ = 2.562 x 10^{22} atoms

But 1 atom releases 2.81 x 10^{-11}J

∴ 2.562 x 10^{22} atoms release 2.81 x 10^{-11} x 2.562 x 10^{22} = 7.1992 x 10^{11}J

Example 29

Given that $^{235}_{92}U$ = 238.12492U, $^{234}_{90}Th$ = 234.1165U, $^{4}_{2}He$ = 4.0038U, 1U = 933MeV

(i) Show that the nucleus of uranium can disintegrate by releasing an alpha particle

$^{235}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

Total energy on the RHS = 234.1165 + 4.0038 = 238 .1203U

Since there is a loss in mass, the reaction is possible and the loss in mass in mass is the energy released.

(ii) Calculate the energy released in the process

Energy released = 238.12492 - 238 .1203

= 0.00462U

=0.00462 x 933MeV

= 4.31046MeV

(iii) Calculate the kinetic energy gained by the alpha particle.

Let Q = total energy released

Q = $K.E_{Th} + K.E_{\alpha}$

= $\frac{1}{2}m_{Th}v_{Th}^2 + \frac{1}{2}m_{\alpha}v_{\alpha}^2$ ……………………………… (i)

From conservation of momentum

Initial momentum = final momentum

$0 = m_{Th}v_{Th} + m_{\alpha}v_{\alpha}$

$v_{Th} = -\frac{m_{\alpha}v_{\alpha}}{m_{Th}}$ ……………………………………………… (ii)

Substitute Eqn. (ii) into Eqn. (i)

Q = $\frac{1}{2}m_{Th}\left(-\frac{m_{\alpha}v_{\alpha}}{m_{Th}}\right)^2 + \frac{1}{2}m_{\alpha}v_{\alpha}^2$

= $\frac{1}{2}\frac{m_{\alpha}^2v_{\alpha}^2}{m_{Th}} + \frac{1}{2}m_{\alpha}v_{\alpha}^2$

Factorize $\frac{1}{2}m_{\alpha}v_{\alpha}^2$

Q = $\frac{1}{2}m_{\alpha}v_{\alpha}^2\left(\frac{m_{\alpha}}{m_{Th}} + 1\right)$

= $\frac{1}{2}m_{\alpha}v_{\alpha}^2\left(\frac{m_{\alpha+}\,m_{Th}}{m_{Th}}\right)$

$\frac{1}{2}m_\alpha v_\alpha^2$ = Q$\left(\frac{m_{Th}}{m_{Th}+ m_\alpha}\right)$

Kinetic energy gained by alpha particle

$\frac{1}{2}m_\alpha v_\alpha^2$ = Q$\left(\frac{m_{Th}}{m_{Th}+ m_\alpha}\right)$

$$= 4.31046\left(\frac{234}{234+4}\right)$$

$$= 4.238MeV$$

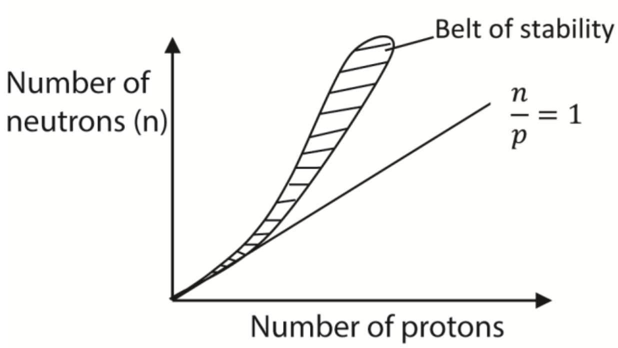
Nuclear stability

The nuclear stability depends on the number of neutrons and protons present in a nucleus.

Stability of light nucleus is most likely when number of protons = number of neutrons, e.g. ${}^{12}_{6}C, {}^{14}_{7}N$…

Stability of heavy nuclei is most likely to occur when the nucleus has more neutrons than protons, e.g. ${}^{206}_{82}Pb$.

A graph of the number of neutrons against the number of protons to show the stability zone



Unstable isotope are found below and above the stability belt. Disintegration produces new isotopes which are closer to stability belt than the original isotopes

Mode of decay

(i) the nuclei above the belt of stability are rich in neutrons and hence disintegrate in such a way that one of the neutron is converted to a proton
i.e. [$^{1}_{0}n \rightarrow ^{1}_{1}H$ or $^{1}_{1}p + _{-1}^{0}e$] or such nuclei emit a β-particle.

Example
(a) $^{24}_{11}Na \rightarrow ^{24}_{12}Mg + _{-1}^{0}e$
(b) $^{14}_{6}C \rightarrow ^{14}_{7}N + _{-1}^{0}e$

(ii) The nuclei lying below the belt of stability are deficient in neutrons and hence disintegrate in such a way that one of their proton is converted into a neutron. The conversion can be done by any of the following two ways
(a) emission of positron: $^{1}_{1}H \rightarrow ^{1}_{0}n + _{+1}^{0}e$
(b) electron capture process
$^{1}_{1}H + _{-1}^{0}e(electron) \rightarrow ^{1}_{0}n$

(iii) $^{208}_{82}Pb$ and $^{209}_{83}Bi$ are the heaviest stable nuclei. Nuclei having higher number of protons or neutrons disintegrate by loss of α ($^{4}_{2}He$), $_{+1}^{0}e$, $_{-1}^{0}e$ or by fission process.

The higher the binding energy the more stable the nucleus.

Binding energy per nucleon is the ratio of energy required to break a nucleus into free neutrons and protons to the mass number.

i.e. binding energy per nucleon = $\frac{Binding\ energy}{Mass\ number}$

Nuclear fission

This is the splitting of a heavy nucleus into two or more light nuclei accompanied by the release of energy.

Sufficient excitation energy for the nucleus to split may be provided by particle bombardment of the nucleus with protons, neutrons or electrons. E.g.

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{57}Mo + ^{139}_{57}La\ 2^{1}_{0}n + 7_{-1}^{0}e$ + energy

Neutrons are preferred as bombarding particles because they do not carry charge and therefore can penetrate deeper into the nucleus.

Uses of nuclear fission

1. To provide electricity
2. To manufacture atomic bombs.

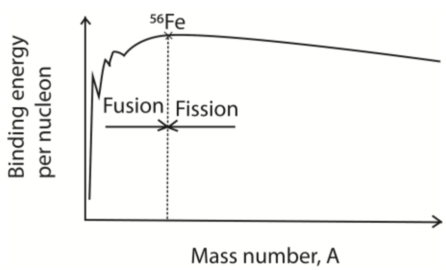
Nuclear fusion

It is the union of two lighter nuclei to produce heavier nucleus of higher binding energy per nucleon accompanied by release of energy, e.g.

$${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He + {}_{0}^{1}n + Energy$$

Nuclear fusion takes place at high temperature because the nuclei need a lot of kinetic energy to overcome their electrostatic repulsion.

A sketch of Variation of Binding energy per nucleon with mass number relating nuclear fusion and nuclear fission



Note that:

- Two Small nuclei with atomic mass less than 56 each fuse to give a heavier nuclei with smaller mass by higher binding energy to increase stability of nucleon
- A nucleus with atomic mass higher than 56 split to form lighter nuclei of higher binding energy per nuclei.